Answer on Question #44021 – Physics – Mechanics-Kinematics-Dynamics

the magnitude of two vectors p bar ad q bar differ by 1. the magnitude of their resultant makes an angle of tan inverse (3/4) with p. the angle between p and q is

Solution:

$\alpha = \arctan\left(\frac{3}{4}\right) -$ angle between vector p and resultant vector;

$\beta -$ angle between vector p and vector q;

First vector:

$$p = (p_x, p_y), \text{magnitude: } P = \sqrt{p_x^2 + p_y^2}$$

Second vector:

$$q = (q_x, q_y), \text{magnitude: } Q = \sqrt{q_x^2 + q_y^2}$$

Let's make a substitution:

$$p_x q_x + p_y q_y = X$$

Difference between magnitude of two vectors:

$$P - Q = 1$$

$$Q = P - 1 \qquad (1)$$

Resultant vector:

$$\vec{r} = \vec{p} + \vec{q}$$

$$r = (p_x + q_x, p_y + q_y)$$

Scalar product of the first vector and resultant vector:

$$\vec{p} \cdot \vec{r} = p_x(p_x + q_x) + p_y(p_y + q_y) = |\vec{p}| \cdot |\vec{r}| \cdot \cos\alpha =$$

$$= P \cdot \sqrt{(p_x + q_x)^2 + (p_y + q_y)^2} \cdot \cos\alpha$$

$$p_x^2 + p_y^2 + p_x q_x + p_y q_y = P \cdot \sqrt{(p_x + q_x)^2 + (p_y + q_y)^2} \cdot \cos\alpha$$

$$P^2 + X = P \cdot \sqrt{(p_x + q_x)^2 + (p_y + q_y)^2} \cdot \cos\alpha =$$

$$= P\sqrt{p_x^2 + 2p_x q_x + q_x^2 + p_y^2 + 2p_y q_y + q_y^2} \cdot \cos\alpha$$

$$= P\sqrt{P^2 + Q^2 + 2X} \cdot \cos\alpha$$

Scalar product of the first vector and second vector:

$$\vec{p} \cdot \vec{q} = p_x q_x + p_y q_y = |\vec{p}| \cdot |\vec{q}| \cdot \cos\beta =$$

$$= P \cdot Q \cdot \cos\beta \qquad (3)$$

$$\cos\beta = \frac{p_x q_x + p_y q_y}{P \cdot Q} = \frac{X}{P \cdot Q}$$

Thus, we have system with three equations:

$$\begin{cases} Q = P - 1 \qquad (1) \\ P^2 + X = P\sqrt{P^2 + Q^2 + 2X} \cdot \cos\alpha \\ \cos\beta = \dfrac{X}{P \cdot Q} \end{cases}$$

We have 3 equations and 4 unknown $(\mathrm{P}, \mathrm{Q}, \mathrm{X}\ \text{and}\ \cos\beta)$, hence we can't find $\cos\beta -$ cosine of $\text{angle between vector p and vector q.}$