Answer on Question #43997-Physics-Mechanics-Kinematics-Dynamics

A young girl gives her toboggan a push of 4.0 m/s up a hill. It slides up the hill slowing down at an acceleration of 8.0 m/s^2 [down]. It comes to a stop and then slides back towards her speeding up at the same rate as it slowed down on the way up. If the girl has to run 48 m down the hill from where it first was pushed to get to where her sled stopped, find the elapsed time for the journey.

Solution

If we say that "down" the hill is the positive direction, then here is the information we have:

$$v = -4.0\frac{m}{s}$$

$$a = 8.0\frac{m}{s^2}$$

$$d = 48\,m$$

Unknown variable is t ("elapsed time for the journey").

So now we need an equation relating $a, t, d, and\ v$:

$$d = \frac{at^2}{2} + vt$$

$$48\,m = 8.0\frac{m}{s^2}\frac{t^2}{2} + \left(-4.0\frac{m}{s}\right)t$$

$$48\,m = t^2 \cdot 4.0\frac{m}{s} - t \cdot 4.0\,m$$

Simplify, and rearrange this to be a standard quadratic equation:

$$t^2 - t - 12 = 0$$

$$(t-4)(t+3) = 0$$

$$t = 4\ or\ t = -3$$

We will pick $t = 4$ seconds, since we are looking for a positive time.

Again, let's make sure this makes sense. She pushed the toboggan up the hill and it slowed to a stop in $\frac{1}{2}$ second $\left(\frac{4.0\frac{m}{s}}{8.0\frac{m}{s^2}}\right)$, slid back down to where she was for another $\frac{1}{2}$ second, and kept going down the hill for three seconds. During those three seconds, it sped up from $4.0\frac{m}{s}\ to\ 4.0 + 3 \cdot 8.0 = 28.0\frac{m}{s}$, so its average speed was $16\frac{m}{s}\ \left(\frac{4+28}{2}\right)$, and it was therefore $48\ m$ down the hill when it ran down the hill and stopped.
Answer: 4 seconds.