

Answer on Question #43941 – Physics – Other

Question.

In an experiment to determine the resistivity of a wire, the following measurements were obtained: resistance = 2.5 ± 0.01 ohms, length = 65.0 ± 0.1 cm, diameter = 0.5 ± 0.01 mm. Given that resistivity is RA/L , calculate: i. the value of resistivity ii. the absolute uncertainty in the resistivity value iii. write the final result with its error.

Given:

$$R \pm dR = 2.5 \pm 0.01 \text{ Ohms}$$

$$L \pm dL = 65 \pm 0.1 \text{ cm} = 0.65 \pm 0.001 \text{ m}$$

$$D \pm dD = 0.5 \pm 0.01 \text{ mm}$$

Find:

- 1) $\rho = ?$
- 2) $d\rho = ?$
- 3) $\rho \pm d\rho = ?$

Solution.

From the given formula resistivity is:

$$\rho = \frac{RA}{L},$$

where $A = \frac{\pi D^2}{4}$ is the cross-sectional area of a wire.

So,

$$\rho = \frac{RA}{L} = \frac{\pi D^2 R}{4L}$$

By first method absolute uncertainty is:

$$d\rho = \frac{\pi (dD)^2 dR}{4dL}$$

1) Calculate:

$$\rho = \frac{\pi D^2 R}{4L} = \frac{3.14 \cdot 0.5^2 \cdot 2.5}{4 \cdot 0.65} = 0.7548 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

2) Calculate:

$$d\rho = \frac{\pi(dD)^2 dR}{4dL} = \frac{3.14 \cdot 0.01^2 \cdot 0.01}{4 \cdot 0.001} = 0.0008 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

3)

$$\rho \pm d\rho = 0.7548 \pm 0.0008 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

It means $\rho \in [0.754; 0.7556]$

But this method doesn't consider the signs of errors. That's why, let use the second method:

1) Calculate:

$$\rho = \frac{\pi D^2 R}{4L} = \frac{3.14 \cdot 0.5^2 \cdot 2.5}{4 \cdot 0.65} = 0.7548 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

2) Calculate:

$$\begin{aligned} \rho - d\rho &= \frac{\pi(D - dD)^2 (R - dR)}{4(L - dL)} = \frac{3.14 \cdot (0.5 - 0.01)^2 \cdot (2.5 - 0.01)}{4 \cdot (0.65 - 0.001)} = \\ &= 0.7231 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}} \end{aligned}$$

$$\begin{aligned} \rho + d\rho &= \frac{\pi(D + dD)^2 (R + dR)}{4(L + dL)} = \frac{3.14 \cdot (0.5 + 0.01)^2 \cdot (2.5 + 0.01)}{4 \cdot (0.65 + 0.001)} = \\ &= 0.7871 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}} \end{aligned}$$

$$d\rho = \frac{1}{2} [(\rho + d\rho) - (\rho - d\rho)] = \frac{1}{2} (0.7871 - 0.7231) = 0.032 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

3)

$$\rho \pm d\rho = 0.7548 \pm 0.032 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

It means $\rho \in [0.7228; 0.7866]$

Answer.

1)

$$\rho = 0.7548 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

2)

$$d\rho = 0.032 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

3)

$$\rho \pm d\rho = 0.7548 \pm 0.032 \frac{\text{Ohm} \cdot \text{mm}^2}{\text{m}}$$

