Answer on Question #43809 – Physics - Mechanics | Kinematics | Dynamics

For moving particle, the relation between time and position is given by t = Ax^2+Bx. Where A and B are constants. Find out the acceleration of the particle as a function of velocity.

Solution:

Relation between time and position:

$$\mathrm{t} = \mathrm{Ax}^2 + \mathrm{Bx}$$

The derivative of a function representing the position of a particle along a line at time t is the instantaneous velocity at that time. The derivative of the velocity, which is the second derivative of the position function, represents the instantaneous acceleration of the particle at time t.

Derivative of of the both parts of the initial equation:

$$\mathrm{dt} = \frac{\mathrm{d}(\mathrm{Ax}^2 + \mathrm{Bx})}{\mathrm{dx}}$$

$$\mathrm{dt} = (2\mathrm{Ax} + \mathrm{B})\mathrm{dx}$$

$$(2\mathrm{Ax} + \mathrm{B})\frac{\mathrm{dx}}{\mathrm{dt}} = 1$$

The velocity of the particle is:

$$\mathrm{v} = \frac{\mathrm{dx}}{\mathrm{dt}} = \frac{1}{2\mathrm{Ax} + \mathrm{B}} = (2\mathrm{Ax} + \mathrm{B})^{-1}$$

The acceleration of the particle is:

$$\mathrm{a} = \mathrm{v}' = \frac{\mathrm{dv}}{\mathrm{dt}} = \frac{\mathrm{d}((2\mathrm{Ax} + \mathrm{B})^{-1})}{\mathrm{dt}} = -(2\mathrm{Ax} + \mathrm{B})^{-2} \cdot 2\mathrm{A}\frac{\mathrm{dx}}{\mathrm{dt}} = -\mathrm{v}^2 \cdot 2\mathrm{Av} = -2\mathrm{Av}^3$$

Answer: acceleration of the particle is $\mathrm{a} = -2\mathrm{Av}^3$