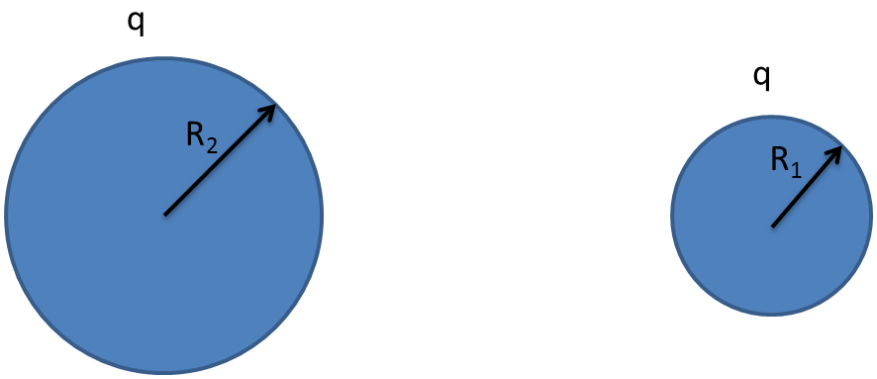
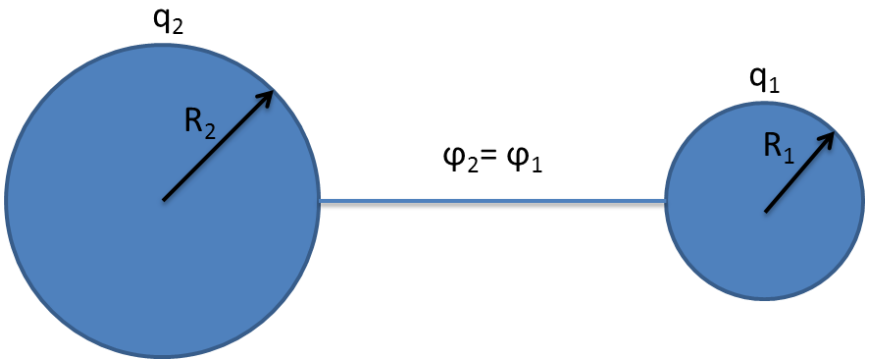
Answer on Question #43684, Physics, Electric Circuits

the radii of two charged spheres are 5cm and 10cm.each has a charge of 75 microcoulomb.they are connected by a conducting wire.calculate their common potential and amount of charge transferred.

Solution.



After connection potentials became equal:



1) The Law of Conservation of Charge:

$$2q = q_1 + q_2$$

2) Potentials are equal:

$$\varphi_1 = \varphi_2 \Rightarrow \frac{1}{4\pi\epsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{R_2} \Rightarrow \frac{q_1}{R_1} = \frac{q_2}{R_2} \Rightarrow q_2 = q_1\frac{R_2}{R_1}$$

So:

$$2q = q_1 + q_1\frac{R_2}{R_1} = q_1\left(1 + \frac{R_2}{R_1}\right) \Rightarrow q_1 = \frac{2q}{1 + \frac{R_2}{R_1}}$$

$$q_2 = 2q - q_1 = 2q - \frac{2q}{1+\frac{R_2}{R_1}} = \frac{2q\left(1+\frac{R_2}{R_1}\right) - 2q}{1+\frac{R_2}{R_1}} = \frac{2q\frac{R_2}{R_1}}{1+\frac{R_2}{R_1}} = \frac{2q}{1+\frac{R_1}{R_2}}$$

In our case $R_2 > R_1$ so $q_1 < q, q_2 > q$

If amount of transferred charge is Δq, then

$q_1 = q - \Delta q;\ q_2 = q + \Delta q$, and:

$$\Delta q = \frac{q_2 - q_1}{2} = \frac{2q}{2}\left(\frac{1}{1+\frac{R_1}{R_2}} - \frac{1}{1+\frac{R_2}{R_1}}\right) = q\left(\frac{\frac{R_2}{R_1}-1}{1+\frac{R_2}{R_1}}\right) = q\frac{\frac{R_2}{R_1}-1}{\frac{R_2}{R_1}+1}$$

Potential:

$$\varphi_1 = \varphi_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\epsilon_0}\frac{2q}{R_1\left(1+\frac{R_2}{R_1}\right)} = \frac{1}{2\pi\epsilon_0}\frac{q}{R_1+R_2}$$

Numerically:

$$\Delta q = 75\cdot 10^{-6}C \cdot \frac{\frac{0.1m}{0.05m}-1}{\frac{0.1m}{0.05m}+1} = 25\cdot 10^{-6}C$$

$$\varphi = \frac{1}{2\cdot 3.14\cdot 8.85\cdot 10^{-12}\frac{F}{m}}\frac{75\cdot 10^{-6}C}{0.05m+0.1m} \approx 9\cdot 10^{6}V$$