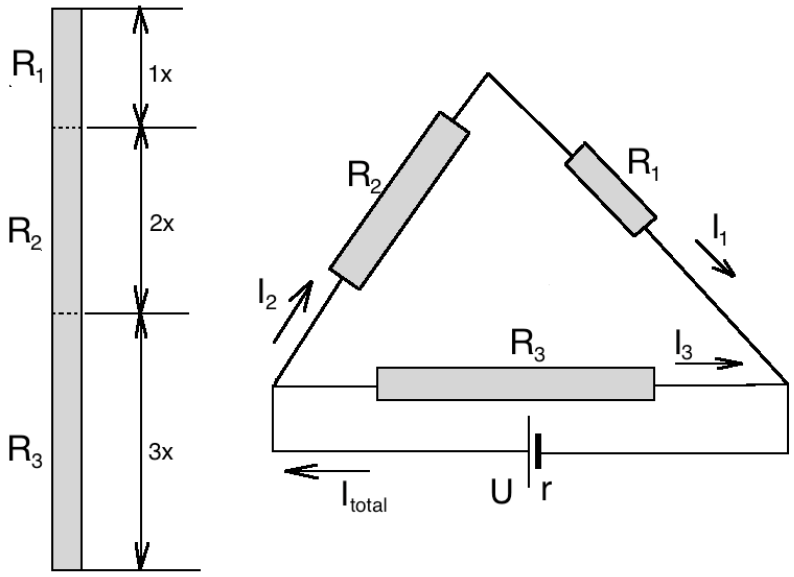
Answer on Question #43573 – Physics - Electric Circuits

a uniform wire of resistance 15ohms is cut into three pieces in ratio 1:2:3 and the three pieces are connected to form a triangle. a cell of 10V potential with internal resistance 1ohm is connected across the highest of three resistors. calculate the current through each part of the circuit.

Solution:



$\mathrm{R} = 15\ \mathrm{Ohm} - \text{resistance of the uniform wire;}$
$1\mathrm{x}:\ 2\mathrm{x}:\ 3\mathrm{x} - \text{ratio of the three pieces;}$
$\mathrm{U} = 10\mathrm{V} - \text{potential of the cell;}$
$\mathrm{r} = 1\ \mathrm{Ohm} - \text{internal resistance of the cell;}$

Since the resistance is linearly proportional to the length of the wire $(\mathrm{R} = \rho\frac{\mathrm{L}}{\mathrm{S}}, \mathrm{L} - \text{length})$, the ratio of the three resistances of the pieces is equal to the ratio of the lengthes of the pieces:

$$\mathrm{R_1}:\mathrm{R_2}:\mathrm{R_3} = 1:2:3$$
$$\mathrm{R_3} = 3\mathrm{R_1}$$
$$\mathrm{R_2} = 2\mathrm{R_1}$$

The sum of these resistances is equal to the resistance of the uniform ware. We have an equation:

$$\mathrm{R} = \mathrm{R_1} + \mathrm{R_2} + \mathrm{R_3}$$
$$\mathrm{R} = \mathrm{R_1} + 2\mathrm{R_1} + 3\mathrm{R_1}$$
$$\mathrm{R} = 6\mathrm{R_1}$$
$$\mathrm{R_1} = \frac{\mathrm{R}}{6};\ \mathrm{R_2} = \frac{2\mathrm{R}}{6};\ \mathrm{R_3} = \frac{3\mathrm{R}}{6} = \frac{\mathrm{R}}{2}$$

In a series configuration, the current through all of the resistors is the same, and resultant resistance is equal to the sum of the resistances:

$$R_{12} = R_1 + R_2 = \frac{R}{6} + \frac{2R}{6} = \frac{3R}{6} = \frac{R}{2}$$

Formula for the resultant resistance for the parallel configuration:

$$\frac{1}{R_{total}} = \frac{1}{R_3} + \frac{1}{R_{12}}$$

$$R_{total} = \frac{R_3 R_{12}}{R_3 + R_{12}} = \frac{R_3 R_{12}}{R_3 + R_{12}} = \frac{\frac{R}{2} \cdot \frac{R}{2}}{\frac{R}{2} + \frac{R}{2}} = \frac{\frac{R^2}{4}}{R} = \frac{R}{4}$$

Formula for the total current through the circuit (Ohm's law):

$$I_{total} = \frac{U}{R_{total} + r} = \frac{U}{\frac{R}{4} + r} = \frac{10V}{\frac{15\ Ohm}{4} + 1\ Ohm} = 2.1\ A$$

Resistors $R_3 = \frac{R}{2}$ and $R_{12} = \frac{R}{2}$ in a parallel configuration are each subject to the same potential difference (voltage), however the currents through them add, since $R_3 = R_{12} \Longrightarrow I_3 = I_2 = \frac{I_{total}}{2} = \frac{2.105\ A}{2} = 1.05\ A$ (current is halved).

In a series configuration (resistors R_1 and R_2), the current through all of the resistors is the same:

$$I_1 = I_2 = 1.05\ A$$

Answer: $I_1 = I_2 = I_3 = 1.05\ A$;
$I_{total} = 2.1\ A$