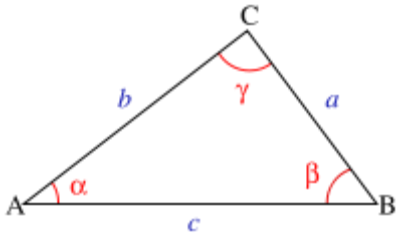
Answer on Question #43510, Physics, Other

What is the angle between two vectors of equal magnitude such that the resultant is one-third as either of the original forces?

Solution:

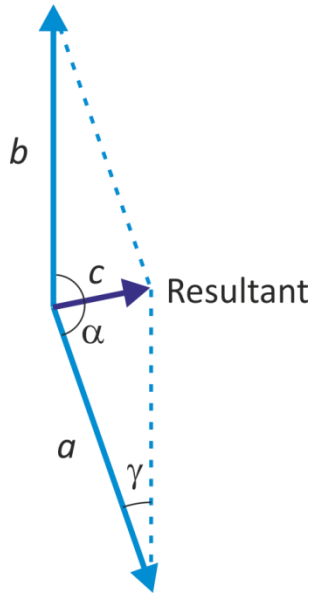
I give the formula for the Law of Cosines and use it to find the missing side length of a triangle.

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$



$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

From given $a = b$, and $c = a/3$.



Thus,

$$\cos\gamma = \frac{a^2 + a^2 - a^2/9}{2aa} = \frac{\frac{17a^2}{9}}{2a^2} = \frac{17}{18}$$

$$\gamma = \cos^{-1}\left(\frac{17}{18}\right) = 19.19°$$

The angle between two vectors is

$$2\alpha = 180° - \gamma$$

$$\pi - \cos^{-1}(x) = \cos^{-1}(-x)$$

$$2\alpha = \pi - \cos^{-1}\left(\frac{17}{18}\right) = \cos^{-1}\left(-\frac{17}{18}\right) = 160.81°$$

Answer: $\cos^{-1}\left(-\frac{17}{18}\right) = 160.81°$