Answer on Question #43415, Physics, Other

A roller coaster trolley, with a mass of 875 kg, comes over the top a hill and accelerates down to climb the next hill that is 25.3 m high. At the bottom of the hill, the trolley has a speed of 20.1 m/s. Relative to the top of the hill, the magnitude of the trolley's mechanical energy at the bottom of the hill is?

Solution:

Given:

$m = 875$ kg,

$h = 25.3$ m,

$v_i = 0$ is the initial velocity,

$v_f = 20.1$ m/s is the final velocity,

$E =?$

The initial hill, or the lift hill, is the tallest in the entire ride. As the train is pulled to the top, it gains potential energy, as explained by the equation for potential energy below:

$$U = mgh$$

where U is potential energy, m is mass, g = 9.81 m/s^2 is acceleration due to gravity and h is height above the ground.

As the roller coaster train begins its descent from the lift hill, the stored potential energy converts to kinetic energy, or energy of motion. The faster the train moves, the more kinetic energy the train gains, as shown by the equation for kinetic energy:

$$K = \frac{mv^2}{2}$$

where K is kinetic energy, m is mass, and v is velocity.

Because the mass of a roller coaster car remains constant, if the speed is increased, the kinetic energy must also increase. This means that the kinetic energy for the roller coaster system is greatest at the bottom of the largest downhill slope on the track, typically at the bottom of the lift hill. When the train begins to climb the next hill on the track, the train's kinetic energy is converted back into potential energy, decreasing the train's velocity.

Mechanical energy E is the sum of the potential and kinetic energies of an object:

$$E = U + K$$

Relative to the top of the hill the trolley's potential energy at the bottom of the hill is

$$U = mg(-h) = -875 \cdot 9.81 \cdot 25.3 = -217168.875 \text{ J}$$

The kinetic energy is

$$K = \frac{mv^2}{2} = \frac{875 \cdot 20.1^2}{2} = 176754.375 \text{ J}$$

Thus, the magnitude of the trolley's mechanical energy at the bottom of the hill is

$$E = |U + K| = |-217168.875 + 176754.375| = 40414.5 \text{ J} = 40.4 \cdot 10^3 \text{ J}$$

Answer: $E = 40.4 \cdot 10^3$ J