Answer on Question #43236 – Physics – Electric Circuits

Question.

A wire of resistance 15.0 ohm is bent to form a regular hexagon ABCDEFA. Find the equivalent resistance of the loop between the points (a) A and B, (b) A and C, (c) A and D.

Solution.

In this problem we can imagine the resistance of the wire, as the sum of the resistances of resistors connected in series or in parallel, which are the sides of a regular hexagon.

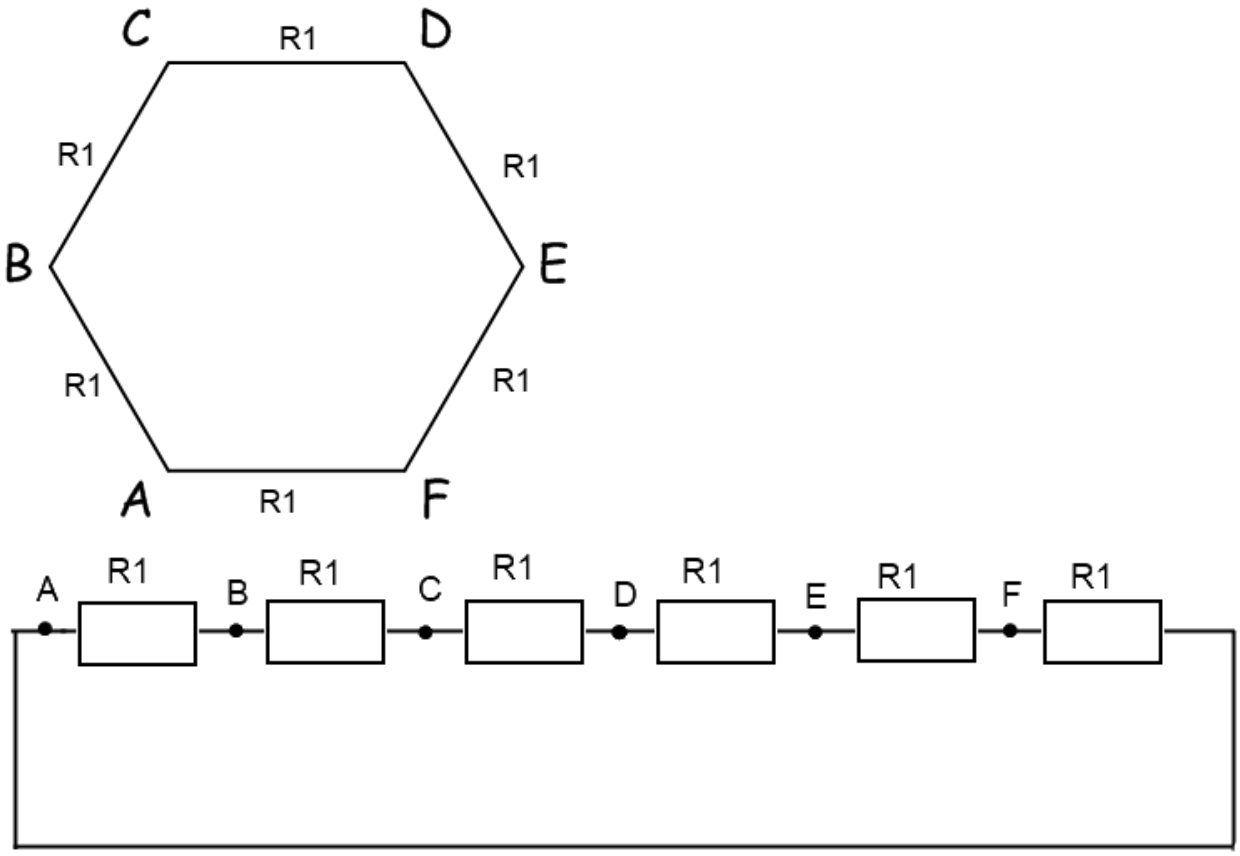

Fig.1. The representation of the sides of a regular hexagon in the form of identical resistors in series.

So,

$R_{total} = 15\ Ohm$ is the resistance of the wire;

$R_1 = R_{AB} = R_{BC} = R_{CD} = R_{DE} = R_{EF} = R_{FA}$.

We can define, that the resistance of each side is equally, because this is a regular hexagon. By definition the resistance is:

$$R = \rho \frac{l}{S}$$

ρ is the electrical resistivity of the material;

l is the length of the material;

S is the cross-sectional area of the material.

For each resistor R_1 we have the same electrical resistivity ρ and cross-sectional area of material S. And the length l is also the same, because a regular hexagon has equal sides.

Thus,

$$R = R_{AB} + R_{BC} + R_{CD} + R_{DE} + R_{EF} + R_{FA} = 6R_1 \rightarrow R_1 = \frac{R}{6} = \frac{15}{6} = 2.5\ Ohm$$

Now let answer the questions:

(a) The resistance between A and B. The equivalent circuit is shown in Fig.2.

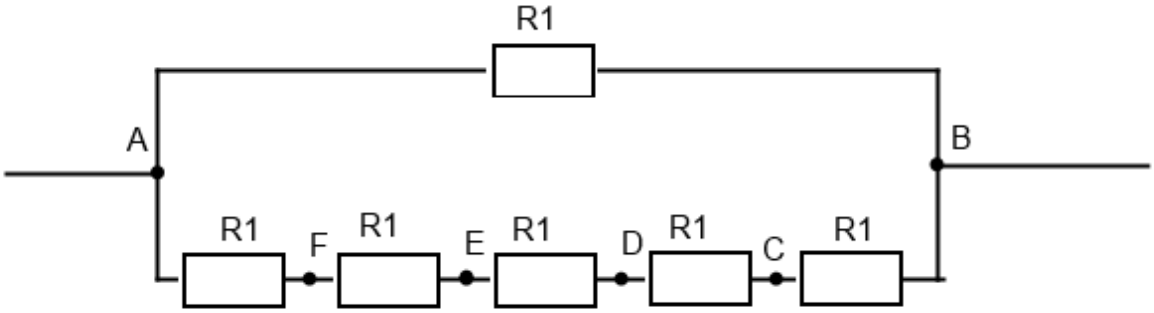

Fig.2. The equivalent resistance of the loop between the points A and B.

$$R_{AB} = \frac{R_1 \cdot 5R_1}{R_1 + 5R_1} = \frac{5}{6}R_1 = \frac{5}{6} \cdot 2.5 = 2.08\ Ohm$$

(b) The resistance between A and C. The equivalent circuit is shown in Fig.3.

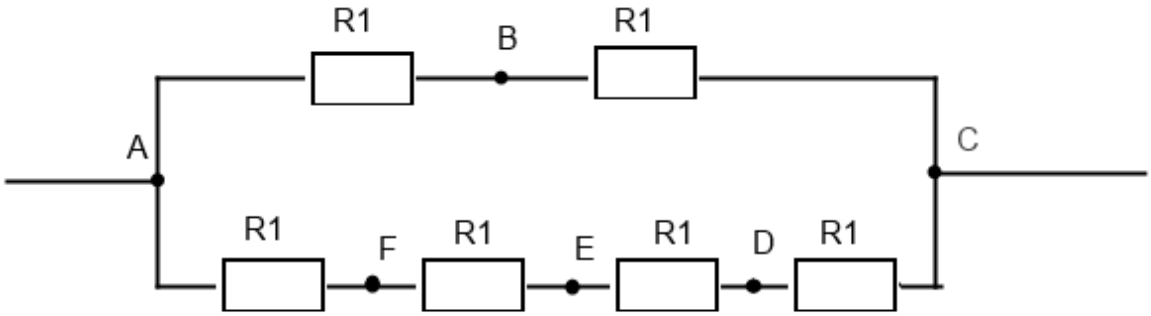

Fig.3. The equivalent resistance of the loop between the points A and C.

$$R_{AC} = \frac{2R_1 \cdot 4R_1}{2R_1 + 4R_1} = \frac{8}{6}R_1 = \frac{8}{6} \cdot 2.5 = 3.33\ Ohm$$

(c) The resistance between A and D. The equivalent circuit is shown in Fig.4.

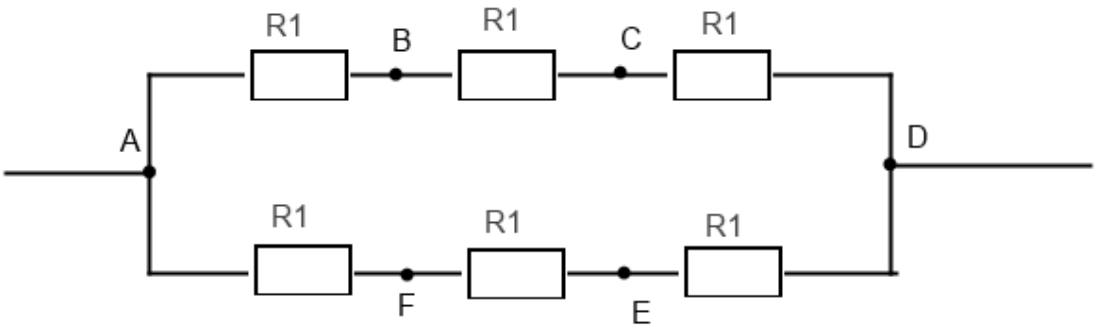

Fig.4. The equivalent resistance of the loop between the points A and D.

$$R_{AD} = \frac{3R_1 \cdot 3R_1}{3R_1 + 3R_1} = \frac{9}{6}R_1 = \frac{3}{2} \cdot 2.5 = 3.75\ Ohm$$

Answer.

(a)

$$R_{AB} = \frac{5}{6}R_1 = 2.08\ Ohm$$

(b)

$$R_{AC} = \frac{8}{6}R_1 = 3.33\ Ohm$$

(c)

$$R_{AD} = \frac{3}{2}R_1 = 3.75\ Ohm$$