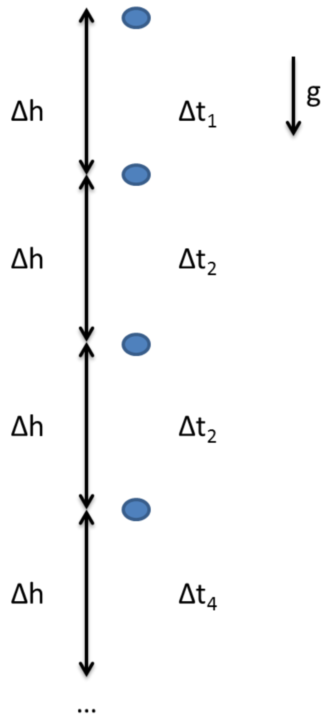
Answer on Question #43152, Physics, Mechanics | Kinematics | Dynamics

A particle is dropped vertically from rest, from a height. Find the time taken by it to fall through successive distances of 1m each.

Solution.



From relations of uniformly accelerated motion:

$$l = V_0 t + \frac{at^2}{2}$$

We have V_0=0, as the particle initially was at the state of rest. So for the first segment we have:

$$\Delta h = \frac{g\Delta t_1^2}{2} \Rightarrow \Delta t_1 = \sqrt{\frac{2\Delta h}{g}}$$

For the first and second segments together:

$$2\Delta h = \frac{g(\Delta t_1 + \Delta t_2)^2}{2} \Rightarrow \Delta t_2 = \sqrt{\frac{2 * 2\Delta h}{g}} - \Delta t_1 = \sqrt{\frac{4\Delta h}{g}} - \sqrt{\frac{2\Delta h}{g}}$$

$$= \sqrt{\frac{2\Delta h}{g}}(\sqrt{2} - 1)$$

For the first three segments together:

$$3\Delta h = \frac{g(\Delta t_1 + \Delta t_2 + \Delta t_3)^2}{2} \Rightarrow \Delta t_3 = \sqrt{\frac{2 * 3\Delta h}{g}} - \Delta t_2 - \Delta t_1$$

$$= \sqrt{\frac{4\Delta h}{g}} - \sqrt{\frac{2\Delta h}{g}}\left(\sqrt{2} - 1\right) - \sqrt{\frac{2\Delta h}{g}} = \sqrt{\frac{2\Delta h}{g}}\left(\sqrt{3} - \sqrt{2}\right)$$

And so on. As one can see from relations forΔt_1, Δt_2 and Δt_3 the relation for interval number i will be next:

$$\Delta t_i = \sqrt{\frac{2\Delta h}{g}}\left(\sqrt{i} - \sqrt{i-1}\right)$$

Numerically:

$$\Delta t_1 = \sqrt{\frac{2\Delta h}{g}} = \sqrt{\frac{2 \cdot 1m}{10\frac{m}{s^2}}} = \sqrt{\frac{1}{5}}s \approx 0.45s$$

$$\Delta t_2 = \sqrt{\frac{2\Delta h}{g}}\left(\sqrt{2} - 1\right) = \sqrt{\frac{2 \cdot 1m}{10\frac{m}{s^2}}}\left(\sqrt{2} - 1\right) \approx 0.19s$$

$$\Delta t_3 = \sqrt{\frac{2\Delta h}{g}}\left(\sqrt{3} - \sqrt{2}\right) = \sqrt{\frac{2 \cdot 1m}{10\frac{m}{s^2}}}\left(\sqrt{3} - \sqrt{2}\right) \approx 0.14s$$

$$\Delta t_i = \sqrt{\frac{2\Delta h}{g}}\left(\sqrt{i} - \sqrt{i-1}\right) = \sqrt{\frac{2 \cdot 1m}{10\frac{m}{s^2}}}\left(\sqrt{i} - \sqrt{i-1}\right) \approx 0.45\left(\sqrt{i} - \sqrt{i-1}\right)s$$

Answer: $\Delta t_i \approx 0.45\left(\sqrt{i} - \sqrt{i-1}\right)s$