

Answer on Question #43123, Physics, Molecular Physics | Thermodynamics

Question.

Two different plates A and B are in composite wall each of dimension 20cm*30cm and their thickness are 3cm and 8cm respectively. Thermal conductivity are 0.25 and 0.85 in C.G.S system.

calculate

1-the thermal resistance of each plate

2-if the free end of plate A at temperature 150c and plate B at temperature 20c calculate the heat current through the combination

3-the temperature of the point of junction

4-the temperature gradient of each plate

Given:

$$a = 20 \text{ cm}$$

$$b = 30 \text{ cm}$$

$$l_1 = 3 \text{ cm}$$

$$l_2 = 8 \text{ cm}$$

$$\lambda_1 = 0.25 \frac{\text{erg}}{\text{s} \cdot \text{cm} \cdot ^\circ\text{C}}$$

$$\lambda_2 = 0.825 \frac{\text{erg}}{\text{s} \cdot \text{cm} \cdot ^\circ\text{C}}$$

Find:

1) $R_1 = ? R_2 = ?$

2) $T_1 = 150^\circ\text{C}; T_2 = 20^\circ\text{C}$
 $q = ?$

3) $T_{junc} = ?$

4) $\nabla T_1 = ? \nabla T_2 = ?$

Solution.

1) By definition thermal resistance is equal to:

$$R = \frac{l}{\lambda S}$$

S is the cross-sectional area. In our case, $S = ab$.

Therefore,

$$R_1 = \frac{l_1}{\lambda_1 ab}; R_2 = \frac{l_2}{\lambda_2 ab}$$

Calculate:

$$R_1 = \frac{3}{0.25 \cdot 20 \cdot 30} = 0.02 \frac{s \cdot K}{erg}$$
$$R_2 = \frac{8}{0.85 \cdot 20 \cdot 30} = 0.0157 \frac{s \cdot K}{erg}$$

3) The heat current density is the amount of energy that flows through a unit area per unit time:

$$q = q_1 = q_2$$

The Fourier equation for thermal conduction:

$$\vec{q} = -\lambda \nabla T$$

In our case,

$$q = \lambda \frac{T_0 - T}{l}$$
$$q_1 = \lambda_1 \frac{T_1 - T_{junc}}{l_1}; q_2 = \lambda_2 \frac{T_{junc} - T_2}{l_2}$$
$$q_1 = q_2 \rightarrow \lambda_1 \frac{T_1 - T_{junc}}{l_1} = \lambda_2 \frac{T_{junc} - T_2}{l_2}$$
$$T_{junc}(l_1 + l_2) = l_1 T_2 + l_2 T_1 \rightarrow T_{junc} = \frac{\lambda_1 l_2 T_1 + \lambda_2 l_1 T_2}{\lambda_2 l_1 + \lambda_1 l_2}$$

Calculate:

$$T_{junc} = \frac{0.25 \cdot 8 \cdot 150 + 0.85 \cdot 3 \cdot 20}{0.85 \cdot 3 + 0.25 \cdot 8} = \frac{351}{4.55} = 77.14^\circ\text{C}$$

2) The heat current through the combination:

$$q = q_1 = \lambda_1 \frac{T_1 - T_{junc}}{l_1} = q_2 = \lambda_2 \frac{T_{junc} - T_2}{l_2}$$

Calculate:

$$q = q_1 = 0.25 \frac{150 - 77.14}{3} = 6.07 \frac{erg}{s \cdot cm^2}$$

4) Calculate the temperature gradient of each plate:

$$\nabla T_1 = -\frac{T_1 - T_{junc}}{l_1}; \nabla T_2 = -\frac{T_{junc} - T_2}{l_2}$$

On other hand,

$$\nabla T_1 = -\frac{q}{\lambda_1}; \nabla T_2 = -\frac{q}{\lambda_2}$$

Calculate:

$$\nabla T_1 = -\frac{150 - 77.14}{3} = -24.28 \frac{^\circ\text{C}}{cm} \text{ or } \nabla T_1 = -\frac{6.07}{0.25} = -24.28 \frac{^\circ\text{C}}{cm}$$

$$\nabla T_2 = -\frac{77.14-20}{8} = -7.14 \frac{^{\circ}\text{C}}{\text{cm}} \text{ or } \nabla T_1 = -\frac{6.07}{0.85} = -7.14 \frac{^{\circ}\text{C}}{\text{cm}}$$

Answer.

1)

$$R_1 = \frac{l_1}{\lambda_1 ab} = 0.02 \frac{\text{s} \cdot \text{K}}{\text{erg}}$$

$$R_2 = \frac{l_2}{\lambda_2 ab} = 0.0157 \frac{\text{s} \cdot \text{K}}{\text{erg}}$$

2)

$$q = q_1 = \lambda_1 \frac{T_1 - T_{junc}}{l_1} = q_2 = \lambda_2 \frac{T_{junc} - T_2}{l_2} = 6.07 \frac{\text{erg}}{\text{s} \cdot \text{cm}^2}$$

3)

$$T_{junc} = \frac{\lambda_1 l_2 T_1 + \lambda_2 l_1 T_2}{\lambda_2 l_1 + \lambda_1 l_2} = 77.14^{\circ}\text{C}$$

4)

$$\nabla T_1 = -\frac{T_1 - T_{junc}}{l_1} = -\frac{q}{\lambda_1} = -24.28 \frac{^{\circ}\text{C}}{\text{cm}}$$

$$\nabla T_2 = -\frac{T_{junc} - T_2}{l_2} = -\frac{q}{\lambda_2} = -7.14 \frac{^{\circ}\text{C}}{\text{cm}}$$