Answer on Question #43116 – Physics – Molecular Physics|Thermodynamics

Question.

Calculate the root mean square velocity of molecule of hydrogen gas at room temperature 27c and calculate the mean kinetic energy of one molecule and one mole.

Given:

$T = 27℃ = 300\ K$

$m(H_2) = 3.34 \cdot 10^{-27}\ kg$

$M(H_2) = 2 \cdot 10^{-3}\ \frac{kg}{mole}$

$\nu = 1\ mole$

Find:

$\bar{v} = ?$

$E_k(1\ molecule) = ?$

$E_k(1\ mole) = ?$

Solution.

From thermodynamics it's known that the average kinetic energy of the gas is proportional to temperature:

$$E_k = \frac{3}{2}kT$$

$k = 1.38 \cdot 10^{-23}\ \frac{J}{K}$ is the Boltzmann's constant;

T is the temperature.

$R = k \cdot N_a$

$R = 8.31\ \frac{J}{mole \cdot K}$ the gas constant;

$N_a = 6.02 \cdot 10^{23}\ mole^{-1}$ is the Avogadro constant.

So, the kinetic energy of $1\ mole$ is:

$$E_k = \frac{3}{2}kT \cdot N_a = \frac{3}{2}RT$$

On the other hand, from classical mechanics we know that the kinetic energy of a particle is:

$$E_k = \frac{m}{2}\bar{v}^2$$

Therefore, the kinetic energy of N particles is:

$$E_k = N\frac{m}{2}\bar{v}^2$$

There N_a molecules in 1 $mole$ of the substance→ $N = N_a$.

The kinetic energy of 1 $mole$ is:

$$E_k = N_a\frac{m}{2}\bar{v}^2$$

So,

$$E_k = N_a\frac{m}{2}\bar{v}^2 = \frac{3}{2}RT \rightarrow \bar{v}^2 = \frac{3RT}{mN_a}$$

$$\nu = \frac{m}{M} = \frac{N}{N_a}$$

ν is the amount of substance;

M is the molar mass.

If $\nu = 1\ mole$, then $M = mN_a$.

Thus,

$$\bar{v}^2 = \frac{3RT}{M} \rightarrow \bar{v} = \sqrt{\frac{3RT}{M}}$$

Let calculate all desired values.

The root mean square velocity is:

$$\bar{v} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \cdot 8.31 \cdot 300}{2 \cdot 10^{-3}}} = 1934\ \frac{m}{s}$$

The kinetic energy of 1 molecule:

$$E_k(1\ molecule) = \frac{m}{2}\bar{v}^2 = \frac{3}{2}kT = \frac{3}{2}1.38 \cdot 10^{-23} \cdot 300 = 6.21 \cdot 10^{-21} J$$

The kinetic energy of 1 $mole$:

$$E_k(1\ mole) = N_a E_k(1\ molecule) = N_a\frac{m}{2}\bar{v}^2 = \frac{3}{2}RT = \frac{3}{2}8.31 \cdot 300 = 3739.5\,J$$

Answer.

The root mean square velocity is:

$$\bar{v} = \sqrt{\frac{3RT}{M}} = 1934\ \frac{m}{s}$$

The kinetic energy of 1 molecule:

$$E_k(1\ molecule) = \frac{m}{2}\bar{v}^2 = \frac{3}{2}kT = 6.21 \cdot 10^{-21} J$$

The kinetic energy of 1 $mole$:

$$E_k(1\ mole) = N_a E_k(1\ molecule) = N_a \frac{m}{2}\bar{v}^2 = \frac{3}{2}RT = 3739.5\ J$$