Answer on Question #43115 – Physics – Molecular Physics | Thermodynamics

Question.

a quantity 4.3 liter of an ideal gas at pressure 2atm is compressed adiabatically to volume 1 liter find
1-the final pressure
2-work done the gas
take gamma constant is 1.4

Given:

$V_0 = 4.3\ l$ is the initial volume

$P_0 = 2\ atm$ is the initial pressure

$V = 1\ l$ is the final volume

$\gamma = 1.4$ is the adiabatic constant

Find:

1) $P = ?$ the final pressure
2) $A = ?$ the work done the gas

Solution.

1) The adiabatic process equation:

$$PV^\gamma = const$$

So,

$$P_0 V_0^\gamma = PV^\gamma \to P = P_0 \left(\frac{V_0}{V}\right)^\gamma$$

Calculate:

$$P = 2 \cdot \left(\frac{4.3}{1}\right)^{1.4} = 2 \cdot 7.7 = 15.4\ atm$$

2) By definition work done is:

$$A = \int_{V_0}^{V} PdV$$

In our case,

$$P = \frac{const}{V^\gamma}$$

But, $const = P_0 V_0^\gamma$. Therefore,

$$P = \frac{P_0 V_0^\gamma}{V^\gamma}$$

Calculate the integral to define the work done:

$$A = \int_{V_0}^{V} PdV = P_0 V_0^\gamma \int_{V_0}^{V} \frac{dV}{V^\gamma} = P_0 V_0^\gamma \frac{1}{1-\gamma} \frac{1}{V^{\gamma-1}} \Big|_{V_0}^{V} = \frac{P_0 V_0^\gamma}{1-\gamma} \left(\frac{1}{V^{\gamma-1}} - \frac{1}{V_0^{\gamma-1}} \right) =$$

$$= \frac{P_0 V_0}{1-\gamma} \left(\left(\frac{V_0}{V} \right)^{\gamma-1} - 1 \right)$$

Calculate:

$$A = \frac{2 \cdot 4.3}{1-1.4} \left(\left(\frac{4.3}{1} \right)^{1.4-1} - 1 \right) = -\frac{8.6}{0.4}(7.7-1) = -21.5 \cdot 6.7 = -144\ atm \cdot l$$

$$1\ atm = 101300\ Pa; 1l = 10^{-3}\ m^3$$

$$A = -144\ atm \cdot l = -144 \cdot 101300 \cdot 10^{-3} = 14587\ Pa \cdot\ m^3 = 14587\ J = 14.587\ kJ$$

Answer.

1)

$$P = P_0 \left(\frac{V_0}{V} \right)^\gamma = 15.4\ atm$$

2)

$$A = \frac{P_0 V_0}{1-\gamma} \left(\left(\frac{V_0}{V} \right)^{\gamma-1} - 1 \right) = 14587\ J = 14.587\ kJ$$