Answer on Question #43048, Physics, Molecular Physics | Thermodynamics

Question.

Two different plates A and B are in composite wall each of dimension 20cm*30cm and their thickness are 3cm and 8cm respectively. Thermal conductivity are 0.25 and 0.85 in C.G.S system.

calculate
1-the thermal resistance of each plate

2-if the free end of plate A at temperature 150c and plate B at temperature 20c calculate the heat current through the combination

3-the temperature of the point of junction

4-the temperature gradient of each plate

Given:

$a = 20\ cm$

$b = 30\ cm$

$l_1 = 3\ cm$

$l_2 = 8\ cm$

$\lambda_1 = 0.25\ \frac{erg}{s \cdot cm \cdot ℃}$

$\lambda_2 = 0.825\ \frac{erg}{s \cdot cm \cdot ℃}$

Find:

1) $R_1 = ?\ R_2 = ?$
2) $T_1 = 150℃;\ T_2 = 20℃$
   $q = ?$
3) $T_{junc} = ?$
4) $\nabla T_1 = ?\ \nabla T_2 = ?$

Solution.

1) By definition thermal resistance is equal to:

$$R = \frac{l}{\lambda S}$$

S is the cross-sectional area. In our case, $S = ab$.
Therefore,

$$R_1 = \frac{l_1}{\lambda_1 ab};\ R_2 = \frac{l_2}{\lambda_2 ab}$$

Calculate:

$$R_1 = \frac{3}{0.25 \cdot 20 \cdot 30} = 0.02\ \frac{s \cdot K}{erg}$$
$$R_2 = \frac{8}{0.85 \cdot 20 \cdot 30} = 0.0157\ \frac{s \cdot K}{erg}$$

3) The heat current density is the amount of energy that flows through a unit area per unit time:

$$q = q_1 = q_2$$

The Fourier equation for thermal conduction:

$$\vec{q} = -\lambda \nabla T$$

In our case,

$$q = \lambda \frac{T_0 - T}{l}$$
$$q_1 = \lambda_1 \frac{T_1 - T_{junc}}{l_1};\ q_2 = \lambda_2 \frac{T_{junc} - T_2}{l_2}$$
$$q_1 = q_2 \rightarrow \lambda_1 \frac{T_1 - T_{junc}}{l_1} = \lambda_2 \frac{T_{junc} - T_2}{l_2}$$
$$T_{junc}(l_1 + l_2) = l_1 T_2 + l_2 T_1 \rightarrow T_{junc} = \frac{\lambda_1 l_2 T_1 + \lambda_2 l_1 T_2}{\lambda_2 l_1 + \lambda_1 l_2}$$

Calculate:

$$T_{junc} = \frac{0.25 \cdot 8 \cdot 150 + 0.85 \cdot 3 \cdot 20}{0.85 \cdot 3 + 0.25 \cdot 8} = \frac{351}{4.55} = 77.14℃$$

2) The heat current through the combination:

$$q = q_1 = \lambda_1 \frac{T_1 - T_{junc}}{l_1} = q_2 = \lambda_2 \frac{T_{junc} - T_2}{l_2}$$

Calculate:

$$q = q_1 = 0.25 \frac{150 - 77.14}{3} = 6.07\ \frac{erg}{s \cdot cm^2}$$

4) Calculate the temperature gradient of each plate:

$$\nabla T_1 = -\frac{T_1 - T_{junc}}{l_1}; \nabla T_2 = -\frac{T_{junc} - T_2}{l_2}$$

On other hand,

$$\nabla T_1 = -\frac{q}{\lambda_1};\ \nabla T_2 = -\frac{q}{\lambda_2}$$

Calculate:

$\nabla T_1 = -\frac{150-77.14}{3} = -24.28 \frac{℃}{cm}$ or $\nabla T_1 = -\frac{6.07}{0.25} = -24.28\ \frac{℃}{cm}$

$\nabla T_2 = -\frac{77.14-20}{8} = -7.14\frac{℃}{cm}$ or $\nabla T_1 = -\frac{6.07}{0.85} = -7.14\ \frac{℃}{cm}$

Answer.

1)

$$R_1 = \frac{l_1}{\lambda_1 ab} = 0.02\ \frac{s \cdot K}{erg}$$

$$R_2 = \frac{l_2}{\lambda_2 ab} = 0.0157\ \frac{s \cdot K}{erg}$$

2)

$$q = q_1 = \lambda_1 \frac{T_1 - T_{junc}}{l_1} = q_2 = \lambda_2 \frac{T_{junc} - T_2}{l_2} = 6.07\ \frac{erg}{s \cdot cm^2}$$

3)

$$T_{junc} = \frac{\lambda_1 l_2 T_1 + \lambda_2 l_1 T_2}{\lambda_2 l_1 + \lambda_1 l_2} = 77.14℃$$

4)

$$\nabla T_1 = -\frac{T_1 - T_{junc}}{l_1} = -\frac{q}{\lambda_1} = -24.28\ \frac{℃}{cm}$$

$$\nabla T_2 = -\frac{T_{junc} - T_2}{l_2} = -\frac{q}{\lambda_2} = -7.14\ \frac{℃}{cm}$$