Answer on Question #43017 – Physics – Electric Circuits

Question.

A given mass of copper is made into a wire with a square cross section of side = 2mm. Another wire made with the same mass has a circular cross section of diameter = 2mm. Find the ratio of their electrical resistances.

Given:

$m_1 = m_2 = m$

$\alpha_1 = \alpha_2 = \alpha$

$\rho_1 = \rho_2 = \rho$

$a = 2\ mm$

$d = 2\ mm$

Find:

$\frac{R_1}{R_2} = ?$

Solution.

By definition electrical resistance is:

$$R = \alpha \frac{l}{S}$$

α is the electrical resistivity of the material;

l is the length of the material;

S is the cross-sectional area of the material.

So,

$R_1 = \alpha_1 \frac{l_1}{S_1}$ is the electrical resistance of first wire;

$R_2 = \alpha_2 \frac{l_2}{S_2}$ is the electrical resistance of second wire.

$$\frac{R_1}{R_2} = \frac{l_1/S_1}{l_2/S_2} = \frac{l_1 S_2}{l_2 S_1}$$

By definition mass is:

$$m = \rho V = \rho l S$$

ρ is the density of the material;

V is the volume of the material.

In our case,

$$m_1 = m_2 \to \rho_1 l_1 S_1 = \rho_2 l_2 S_2$$

But $\rho_1 = \rho_2$, therefore:

$$l_1 S_1 = l_2 S_2 \to l_1 = \frac{S_2}{S_1} l_2$$

So,

$$\frac{R_1}{R_2} = \frac{l_1 S_2}{l_2 S_1} = \frac{S_2}{S_1} l_2 \cdot \frac{S_2}{l_2 S_1} = \left(\frac{S_2}{S_1}\right)^2$$

$S_1 = a^2$ is square cross-sectional area;

$S_2 = \frac{1}{4}\pi d^2$ is circular cross-sectional area.

Thus,

$$\frac{R_1}{R_2} = \left(\frac{S_2}{S_1}\right)^2 = \left(\frac{\pi d^2}{4a^2}\right)^2$$

Calculate:

$$\frac{R_1}{R_2} = \left(\frac{\pi \cdot 2^2}{4 \cdot 2^2}\right)^2 = \left(\frac{\pi}{4}\right)^2 = \left(\frac{3.14}{4}\right)^2 = 0.616$$

Answer.

$\frac{R_1}{R_2} = 0.616$