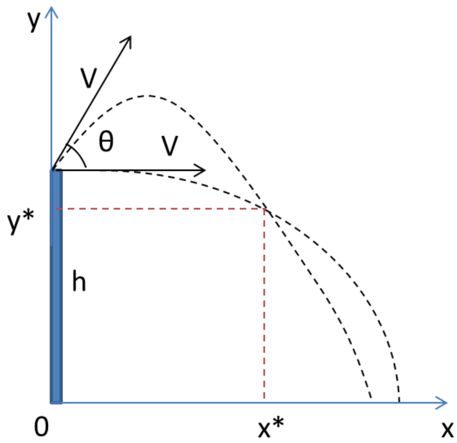
Answer on Question #42893, Physics, Other

21. From the top of height 10 m, one fire is shot horizontally with a speed of $5\sqrt{3}$ m/s. Another fire is shot upwards at angle of 60° with the horizontal at some interval of time with the same speed of $5\sqrt{3}$ m/s. The shots collide in the air at a certain point. The time interval between the two fires is:

Solution.



Assume, two trajectories intersected at point (x*,y*)

Trajectory equations:

$$x = V_x * t$$

$$y = h + V_y * t - \frac{g * t^2}{2}$$

For fire one at point (x*,y*):

$$x^* = V * t_1$$

$$y^* = h - \frac{g * {t_1}^2}{2}$$

For fire two at point (x*,y*):

$$x^* = V * \cos\theta * t_2$$

$$y^* = h + V * \sin\theta * t_2 - \frac{g * {t_2}^2}{2}$$

Thus:

$$V * t_1 = V * \cos\theta * t_2$$

$$h - \frac{g * {t_1}^2}{2} = h + V * \sin\theta * t_2 - \frac{g * {t_2}^2}{2}$$

Next step:

$$\frac{t_1}{t_2} = \cos\theta$$

$${t_1}^2 = {t_2}^2 - \frac{2 * V * \sin\theta * t_2}{g}$$

Then:

$$t_2 - t_1 = (1 - \cos\theta) * t_2$$

$$(\cos\theta)^2 = 1 - \frac{2 * V * \sin\theta}{g * t_2}$$

Then:

$$t_2 = \frac{2 * V * \sin\theta}{g * (1 - cos^2\theta)}$$

$$\Delta t = t_2 - t_1 = (1 - \cos\theta) * \frac{2 * V * \sin\theta}{g * (1 - cos^2\theta)} = \frac{2 * V * \sin\theta}{g * (1 + \cos\theta)}$$

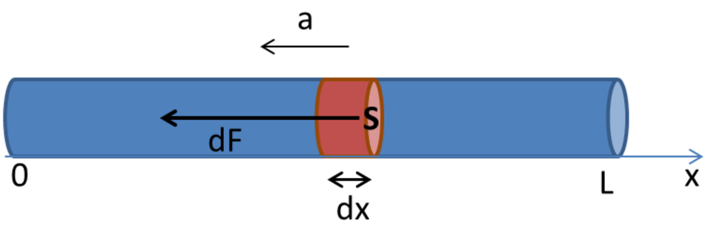
Numerically:

$$\Delta t \approx \frac{2 * 5\sqrt{3} * \frac{\sqrt{3}}{2}}{10 * \left(1 + \frac{1}{2}\right)} s = 1s$$

Answer: (b) 1 s

22. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both ends. The tube is rotated in a horizontal plane about one of its ends with a uniform angular velocity w. The force exerted by the liquid at the other end is:

Solution.



Consider a very thin layer of liquid ***dx*** and mass ***dm***, which has a centripetal acceleration ***a*** due to action of force ***dF.*** The 2nd Newton's law in projection on x axis:

$$dF = dm * a$$

Here:

$$dm = \rho * S * dx$$

$$a = \omega^2 * x$$

Thus:

$$dF = \rho * S * \omega^2 * xdx$$

After integration:

$$F = \int_0^F dF = \int_0^L \rho * S * \omega^2 * xdx = \frac{1}{2}\rho S\omega^2 L^2 = \frac{1}{2} m\omega^2 L$$

Answer: (a) $\frac{1}{2} m\omega^2 L$