Answer on Question #42707 – Physics – Mechanics | Kinematics | Dynamics

## Question.

Water is shooting at the same speed out of the three tubes placed on the ground at different angles of 60°, 45°, and 30° to the horizon. Find the ratio of the greatest heights the streams can reach and the ratio of distances from the mouths of the tubes the points where the streams hit the ground.

$v_{0_1} = v_{0_2} = v_{0_3} = v_0$ all streams have the same velocity $v_0$

$\alpha_1 = 60°$ is the angle to the horizon of tube 1

$\alpha_2 = 45°$ is the angle to the horizon of tube 2

$\alpha_3 = 30°$ is the angle to the horizon of tube 3

$h_{\max 1}: h_{\max 2}: h_{\max 3} = ?$ is the ratio of the greatest heights

$l_{\max 1}: l_{\max 2}: l_{\max 3} = ?$ is the ratio of distances where the streams hit the ground

## Solution.

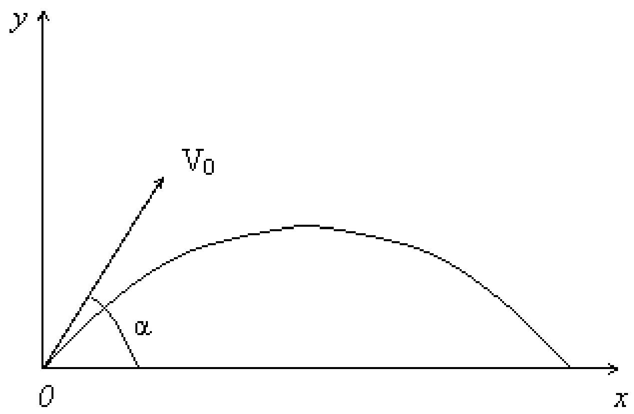


Fig. 1. Motion of body thrown at angle to the horizon.

Problem of the body thrown at an angle to the horizon is known problem and we can see the following:

$\vec{F} = m\vec{a}$ is the Newton's second law

$a_x = 0;\ a_y = g$

$v_x = v_0 \cos\alpha\,;\ v_y = v_0 - gt = v_0 - g\sin\alpha$

$x = v_0 t\cos\alpha;\ y = v_0 t\sin\alpha - \frac{1}{2}gt^2$

Let define the flight time. When the stream hit the ground $y = 0$. Therefore:

$$0 = v_0 t_{all}\sin\alpha - \frac{1}{2}gt_{all}^2 \rightarrow t_{all} = \frac{2v_0\sin\alpha}{g}$$

Graph of flight is symmetric, so the top point (the greatest height) corresponds to the time:

$$t_0 = \frac{t_{all}}{2} = \frac{v_0\sin\alpha}{g}$$

Substitute this value of time $t_0$ into the formula for $y$:

$$y(t_0) = y_{max} = h_{max} = v_0 t_0\sin\alpha - \frac{1}{2}g{t_0}^2 = \frac{v_0^2\sin^2\alpha}{2g}$$

So,

$$h_{\max 1}{:}\,h_{\max 2}{:}\,h_{\max 3} = \sin^2\alpha_1 : \sin^2\alpha_2 : \sin^2\alpha_3$$

Calculate:

$$\sin\alpha_1 = \frac{\sqrt{3}}{2} \rightarrow \sin^2\alpha_1 = \frac{3}{4}$$

$$\sin\alpha_2 = \frac{1}{\sqrt{2}} \rightarrow \sin^2\alpha_2 = \frac{1}{2} = \frac{2}{4}$$

$$\sin\alpha_3 = \frac{1}{2} \rightarrow \sin^2\alpha_3 = \frac{1}{4}$$

$$h_{\max 1}{:}\,h_{\max 2}{:}\,h_{\max 3} = \sin^2\alpha_1 : \sin^2\alpha_2 : \sin^2\alpha_3 = 3{:}\,2{:}\,1$$

We understand that it is correct. The greater the angle, the greater the maximum height.

Now let find the maximum length:

$$l_{max} = x_{max} = x(t_{all}) = v_0 t_{all}\cos\alpha = \frac{2v_0^2\cos\alpha\sin\alpha}{g} = \frac{v_0^2\sin 2\alpha}{g}$$

So,

$$l_{\max 1}{:}\,l_{\max 2}{:}\,l_{\max 3} = \sin 2\alpha_1 : \sin 2\alpha_2 : \sin 2\alpha_3$$

Calculate:

$$\sin 2\alpha_1 = \sin 120° = \sin 60° = \sin\alpha_1 = \frac{\sqrt{3}}{2}$$

$$\sin 2\alpha_2 = \sin 90° = 1$$

$$\sin 2\alpha_3 = \sin 60° = \sin\alpha_1 = \frac{\sqrt{3}}{2}$$

$$l_{\max 1} : l_{\max 2} : l_{\max 3} = \sin 2\alpha_1 : \sin 2\alpha_2 : \sin 2\alpha_3 = \frac{\sqrt{3}}{2} : 1 : \frac{\sqrt{3}}{2}$$

He will fly the longest distance at angle $45°$.

## Answer.

$$h_{\max 1} : h_{\max 2} : h_{\max 3} = \sin^2\alpha_1 : \sin^2\alpha_2 : \sin^2\alpha_3 = 3 : 2 : 1$$

$$l_{\max 1} : l_{\max 2} : l_{\max 3} = \sin 2\alpha_1 : \sin 2\alpha_2 : \sin 2\alpha_3 = \frac{\sqrt{3}}{2} : 1 : \frac{\sqrt{3}}{2}$$