Answer on Question #42638-Physics-Molecular Physics-Thermodynamics

Consider a compound slab consisting of two different materials having equal thickness and thermal conductivities K and 2K. The equivalent conductivity of the slab is

Options are

(a) 2K/3 (b) (2^1/2)K (c) 3K (d) 4K/3

Solution

Let length is L, area is A, T_1 be the temperature of the sink, T_2 be the temperature of the source, T be the temperature of the junction.

For the first conductor, the thermal rate is given by

$$\frac{H_1}{t} = \frac{KA(T - T_1)}{L}.$$

For the second conductor, the thermal rate is given by

$$\frac{H_2}{t} = \frac{2KA(T_2 - T)}{L}.$$

The two slabs are connected in series so the heat current flowing through will be same. So,

$$\frac{KA(T - T_1)}{L} = \frac{2KA(T_2 - T)}{L} \rightarrow (T - T_1) = 2(T_2 - T) \rightarrow \mathrm{T} = \frac{1}{3}(2\mathrm{T}_2 + \mathrm{T}_1).$$

Let the equivalent coefficient of thermal conductivity be K'

Adding thermal rate for the first and second conductors

$$\frac{(H_1 + H_2)}{t} = \frac{KA(T - T_1)}{L} + \frac{2KA(T_2 - T)}{L} \rightarrow \frac{H}{t} = \frac{\left[\mathrm{KA}\left\{\frac{1}{3}(2\mathrm{T}_2 + \mathrm{T}_1) - \mathrm{T}_1\right\} + 2\mathrm{KA}\left\{\mathrm{T}_2 - \frac{1}{3}(2\mathrm{T}_2 + \mathrm{T}_1)\right\}\right]}{\mathrm{L}}$$

$$\frac{H}{t} = \frac{KA\frac{4}{3}(T_2 - T_1)}{L} = \frac{K'A(T_2 - T_1)}{L}.$$

So, equivalent $K' = \frac{4}{3}K$.

Answer: (d) 4K/3.