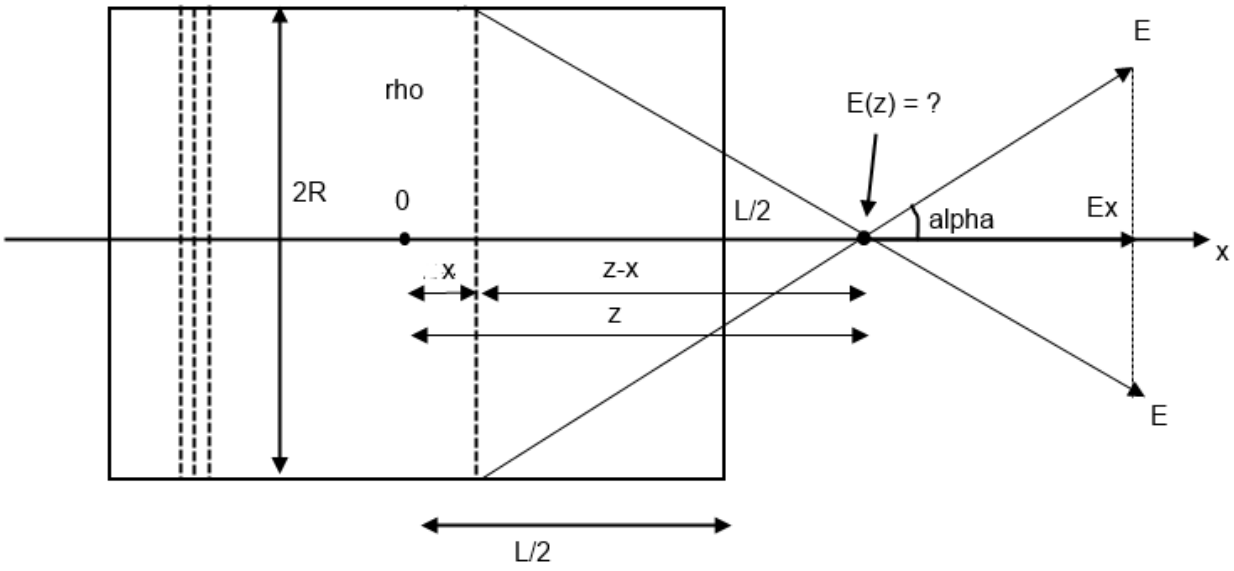
Answer on Question #42494 – Physics – Electromagnetism

Question.

A solid cylinder of length L and radius R has a uniform charge density of rho "P". Calculate the electric field vector on the axis of the cylinder, a distance z from the center, and outside the cylinder. i.e.: z > L/2

Solution.

Let break our half-cylinder into many thin threads:



We know that the field of infinite thin thread is:

$$E = \frac{2\lambda}{r}$$

where λ is the uniformly distributed linear charge,

r is a distance from thread to point. In our case $r \to |z - x|$.

So, electric field of each thread is:

$$dE = \frac{2d\lambda}{|z - x|}$$

Projection on x-axis:

$$dE_x = \frac{2d\lambda}{|z - x|}\cos\alpha$$

If $< \frac{L}{2}$:

$$q = \rho V = \lambda \cdot 2R \to \lambda = \frac{\rho V}{2R} = \frac{\rho \pi R^2}{2R} L = \frac{\rho \pi R}{2} L$$

$$d\lambda = \frac{\rho \pi R}{2} dx$$

$$dE_x = \frac{2d\lambda}{|z - x|} \cos \alpha = \frac{\rho \pi R}{|z - x|} \cos \alpha \, \mathrm{dx}$$

So,

$$E_x = \int dE_x = \int_0^{-\frac{L}{2}+x} \int_0^{Pi} \frac{\rho \pi R}{|z - x|} \cos \alpha \, \mathrm{d}\alpha \, \mathrm{dx} = 2\rho \pi R \cdot \ln \frac{L}{2}$$

If $z > \frac{L}{2}$:

$$q = \rho V = \lambda \cdot 2R \to \lambda = \frac{\rho V}{2R} = \frac{\rho \pi R^2}{2R} L = \frac{\rho \pi R}{2} L$$

$$dE_x = \frac{2\lambda}{|z - L/2|} \cos \alpha \, \mathrm{d}\alpha = \frac{\rho \pi R L}{|z - L/2|} \cos \alpha \, \mathrm{d}\alpha$$

So,

$$E_x = \int dE_x = \int_0^{Pi} \frac{\rho \pi R}{|z - x|} \cos \alpha \, \mathrm{d}\alpha = \frac{2\rho \pi R L}{|z - L/2|}$$

$$E_x(z \to \infty) = 0$$

Answer.

$z < \frac{L}{2} : E_x = 2\rho \pi R \cdot \ln \frac{L}{2}$

$z > \frac{L}{2} : E_x = \frac{2\rho \pi R L}{|z - L/2|}$