Answer on Question #42129-Physics-Mechanics

A piledriver hammer of mass 150 kg falls freely through a distance of 5 m to strike a pile of mass 400 kg and drives it 75 mm into the ground. The hammer does not rebound when driving the pile. Determine the average resistance of the ground.

Compare and contrast the use of D'Alembert's principle with the principle of conservation of energy when solving the problem.

Solution

Finding velocity v1 of the hammer immediately before impact with the pile using equation for linear motion.

$$v_1^2 = u^2 + 2g(h_2 - h_1) \rightarrow v_1 = \sqrt{(0 + (2 \cdot 9.81 \cdot 5))} = \sqrt{98.1} = 9.9\frac{\text{m}}{\text{s}}.$$

Finding velocity of pile immediately after impact using principle of conservation of momentum.

Momentum of system before impact = Momentum of system after impact

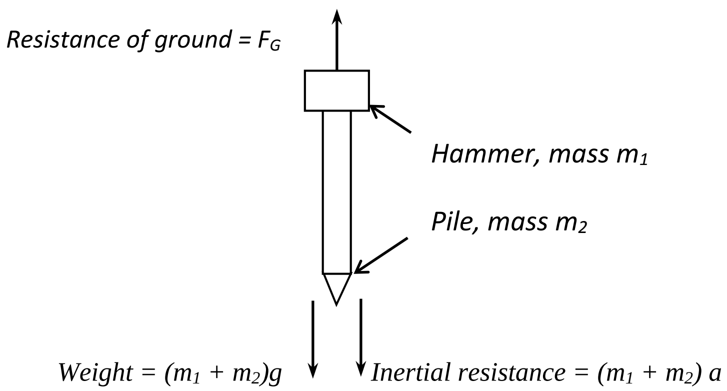
$$m_1 v_1 = (m_1 + m_2)v_2 \rightarrow v_2 = \frac{m_1 v_1}{(m_1 + m_2)} = \frac{150 \cdot 9.9}{(150 + 400)} = 2.7\frac{m}{s}.$$

Finding retardation a, of pile if final velocity, $v_3 = 0$.

$$v_3^2 = v_2^2 + 2a(h_3 - h_2) \rightarrow a = \frac{v_3^2 - v_2^2}{2(h_3 - h_2)} = \frac{0 - 2.7^2}{2 \cdot 0.075} = -48.6\frac{\text{m}}{\text{s}^2}.$$

(The minus sign denotes retardation and can be ignored in the following calculation).

Finding resistance of ground.



$$F_G = (m_1 + m_2)g + (m_1 + m_2)a = (m_1 + m_2)(a + g) = (150 + 400)(9.81 + 48.6) = 32.1 \text{ kN}.$$

Compare and contrast the use of D'Alembert's principle with the principle of conservation of energy to solve an engineering problem.

Finding velocity v1 of the hammer immediately before impact with the pile using principle of conservation of energy.

Loss of PE = Gain of KE

$$m_1 g(h_2 - h_1) = \frac{m_1 v_1^2}{2} \rightarrow v_1 = \sqrt{(2 \cdot 9.81 \cdot 5)} = \sqrt{98.1} = 9.9\,\frac{\text{m}}{\text{s}}.$$

Finding velocity of pile immediately after impact using principle of conservation of momentum.

Momentum of system = Momentum of system
before impact after impact

$$m_1 v_1 = (m_1 + m_2)v_2 \rightarrow v_2 = \frac{m_1 v_1}{(m_1 + m_2)} = \frac{150 \cdot 9.9}{(150 + 400)} = 2.7\,\frac{m}{s}.$$

Finding the average resistance of the ground using principle of conservation of energy.

Work done = Change of PE + Change of KE + Work done against friction

$$W = (m_1 + m_2)g(h_3 - h_2) + \frac{1}{2}(m_1 + m_2)(v_3^2 - v_2^2) + F_G(h_2 - h_3).$$

(Where there is no external work input to the system i.e. $W = 0$).

$$0 = (50 + 400) \cdot 9.81 \cdot (0 - 0.075) + \frac{1}{2} \cdot (150 + 400)\big(0 - 2.7^2\big) + F_G \cdot 0.075 \rightarrow F_G = 32.1\ \text{kN}.$$

In comparing the two methods, it might be concluded that in this particular case there is very little to choose between them. The method used in the first case required 4 separate calculations whereas the energy method used in the second case required only 3. The final one in the second case was however a lengthy calculation. Both methods required use of the principle of conservation of momentum to determine the value of velocity immediately after impact of the hammer. Both methods return identical answers with the same degree of accuracy.