Answer on Question #42081, Physics, Molecular Physics | Thermodynamics

The comet Encke has an aphelion distance of 6.1× 10^{11}m and perihelion distance of 5.1× 10^^{11}m.The mass of the sun is 2.0 ×10^{30}kg.Find the speed of the comet at the perihelion and the aphelion.

Solution:

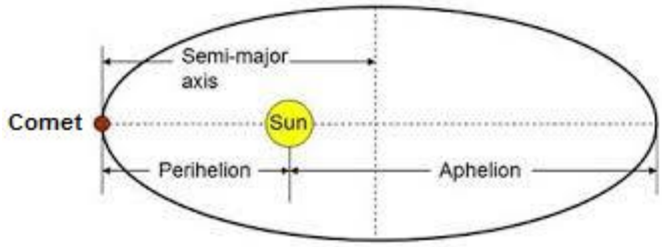
a) Given:

a = aphelion = 6.1× 10^{11} m

b = perihelion = 5.1× 10^{11} m

M = 2.0 × 10^{30} kg

v_a = ?, v_b = ?



The total mechanical energy (TE) of a comet, or any orbiting body, is the sum of its kinetic energy (KE) and its gravitational potential energy (PE):

TE = KE + PE = constant

$$\mathrm{TE} = \frac{mv^2}{2} - m\frac{GM}{r} = constant$$

where M is the mass of the Sun, m is the mass of the comet, r is its instantaneous distance from the Sun and G (6.673 × 10^{-11} Nm^2 kg^{-2}) is the universal gravitational constant.

When PE > KE the comet will have an elliptical orbit with its total mechanical energy given by

$$\mathrm{TE} = -m\frac{GM}{2s}$$

$$-m\frac{GM}{2s} = \frac{mv^2}{2} - m\frac{GM}{r}$$

The comet has a velocity given by

$$v = \sqrt{\frac{GM}{s}\left(\frac{2s}{r} - 1\right)}$$

where s is the mean radius of its orbit (sometimes referred to as the semi-major axis).

The aphelion + perihelion = the major axis.

$$s = \frac{a+b}{2} = 5.6 \times 10^{11}\ \mathrm{m}$$

The speed of the comet at the aphelion

$$v_a = \sqrt{\frac{GM}{s}\left(\frac{2s}{a} - 1\right)} = \sqrt{\frac{6.673 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{5.6 \cdot 10^{11}}\left(\frac{2 \cdot 5.6}{6.1} - 1\right)} = 14115.7\,\frac{\mathrm{m}}{\mathrm{s}} = 14.1\ \mathrm{km/s}$$

The speed of the comet at the perihelion

$$v_b = \sqrt{\frac{GM}{s}\left(\frac{2s}{b} - 1\right)} = \sqrt{\frac{6.673 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{5.6 \cdot 10^{11}}\left(\frac{2 \cdot 5.6}{5.1} - 1\right)} = 16883.5\ \frac{\mathrm{m}}{\mathrm{s}} = 16.9\ \mathrm{km/s}$$

Answer. v_a = 14.1 km/s,
v_b = 16.9 km/s.