Answer on Question #42036-Physics-Mechanics –Kinematics

6. A block of wood floats in freshwater with two-thirds of its volume submerged. In oil the block floats with 0.90 of its volume submerged. Find the density of the wood and the oil.

Solution

An object will float if its weight is equal to the buoyant force on the object. Meanwhile, we have from Archimedes' Principle that the buoyant force on an object is equal to the weight of fluid displaced by the object. So, we can write

$$W_{object} = weight\ of\ water\ displaced \rightarrow (mass\ of\ wood\ block)g = (mass\ of\ water\ displaced)g$$

$$(\rho_{wood} V_{wood})g = (\rho_{water} V_{water\ displaced})g \rightarrow \frac{\rho_{wood}}{\rho_{water}} = \frac{V_{water\ displaced}}{V_{wood}}.$$

Now, since the wood floats with 2/3 of its volume submerged, $V_{water\ displaced} = \frac{2}{3} V_{wood}$, so

$$\rho_{wood} = \rho_{water} \frac{\frac{2}{3} V_{wood}}{V_{wood}} = \frac{2}{3} \rho_{water} = \frac{2}{3} \cdot 1000 \frac{kg}{m^3} = 6.7 \cdot 10^2 \frac{kg}{m^3}.$$

Carrying out the same procedure as above, except with "water" replaced by "oil" we get

$$\frac{\rho_{wood}}{\rho_{oil}} = \frac{V_{oil\ displaced}}{V_{wood}} = \frac{0.9 V_{wood}}{V_{wood}}$$

so that

$$\rho_{oil} = \frac{\rho_{wood}}{0.9} = 7.4 \cdot 10^2 \frac{kg}{m^3}.$$

Answer: $\mathbf{6.7 \cdot 10^2 \frac{kg}{m^3}; 7.4 \cdot 10^2 \frac{kg}{m^3}}$.

7. An iron anchor of density 7870 kg/m3 appears 200 N lighter in water than in air. A) What is the volume of the anchor? B) How much does it weigh in air?

Solution

A) Since the buoyant force is given by

$$F_B = weight\ in\ air - weight\ in\ water$$

and we are told that the object appears 200 N lighter in water than in air, this means that

$$F_B = 200\ N.$$

By Archimedes' Principle,

$$F_B = weight\ of\ water\ displaced = (mass\ of\ water\ displaced)g = (\rho_w V_{wd})g$$

where ρ_w is the density of water, and V_{wd} is the volume of water displaced. Substituting numbers, we get

$$200\ N = \left(1000 \frac{kg}{m^3}\right) V_{wd} \left(9.8 \frac{m}{s^2}\right)$$

so that

$$V_{wd} = \frac{200}{1000 \cdot 9.8} = 2.04 \cdot 10^{-2} m^3.$$

Presumably, the object is completely immersed in water, so it is displacing a volume of water equal to its own volume, i.e., $V = V_{wd}$.

Therefore, the volume of the object is

$$V = \frac{200}{1000 \cdot 9.8} = 2.04 \cdot 10^{-2} m^3.$$

B) To find the weight of the anchor in air, we need to find its mass, which from the definition of density is given by density (of iron, the material of which the anchor is made) times the volume of the anchor which we determined in part A).

So, weight of the anchor is

$$W = (mass\ of\ anchor)g$$

which gives

$$W = \rho_{iron} V g = 7870 \frac{kg}{m^3} \cdot 2.04 \cdot 10^{-2} m^3 \cdot\ 9.8 \frac{m}{s^2}.$$

Therefore

$$W = 1570\ N.$$

Answer: A) $\mathbf{2.04 \cdot 10^{-2} m^3}$; B) $\mathbf{1570\ N}$.