Answer on Question #42033 – Physics – Molecular Physics|Thermodynamics

Question.

What is the resultant temperature when 100 g of steam at 100° C is passed through 500 g of ice at -20° C. the specific heat of water is 0.5 cal g-1° C-1

Given:

$m_1 = 100\ g = 0.1\ kg$ is a mass of steam

$m_2 = 500\ g = 0.5\ kg$ is a mass of ice

$T_1 = 100℃$ is an initial temperature of steam

$T_2 = 0℃$ is an initial temperature of ice

$r = 2.26 \cdot 10^6\ \frac{J}{kg}$ is a heat of vaporization of steam

$\lambda = 0.334 \cdot 10^6\ \frac{J}{kg}$ is a heat of fusion of ice

$c = 1\ \frac{cal}{g \cdot ℃} = 4200\ \frac{J}{kg \cdot ℃}$ is a heat capacity of water

Find:

$T = ?$ is a resultant temperature

Solution.

The heat of condensation of steam and cooling the resulting water to a temperature T is completely the heat of ice melting and heating the resulting water to a temperature T. In this case a mass of vapor m_1 and a mass of ice m_2 form a mass of water $(m_1 + m_2)$.

Write the heat balance equation:

$$Q_1 = Q_2$$

Q_1 is a heat which steam gave

Q_2 is heat which ice got

$$Q_1 = m_1 r + m_1 c(T_1 - T)$$

$m_1 r$ is a heat of condensing, steam converts to water

$m_1 c(T - T_1)$ is a heat of water cooling to the temperature T

$$Q_2 = m_2 \lambda + m_2 c(T - T_2)$$

$m_2\lambda$ is a heat of fusion, ice converts to water

$m_2c(T-T_2)$ is a heat of water warming to the temperature T

So,

$$m_1r + m_1c(T_1 - T) = m_2\lambda + m_2c(T - T_2)$$

$$m_1r + m_1cT_1 - m_1cT = m_2\lambda + m_2cT - m_2cT_2$$

$$(m_1 + m_2)cT = m_1r - m_2\lambda + m_1cT_1 + m_2cT_2$$

Therefore,

$$T = \frac{m_1r - m_2\lambda + m_1cT_1 + m_2cT_2}{(m_1 + m_2)c} = \frac{m_1r - m_2\lambda}{(m_1 + m_2)c} + \frac{m_1T_1 + m_2T_2}{m_1 + m_2}$$

Calculate:

$$T = \frac{0.1 \cdot 2.26 \cdot 10^6 - 0.5 \cdot 0.334 \cdot 10^6}{0.6 \cdot 4200} + \frac{0.1 \cdot 100 + 0.5 \cdot 0}{0.6} = \frac{0.059 \cdot 10^6}{2.52 \cdot 10^3} + \frac{100}{6} =$$
$$= 23.41 + 16.67 = 40.08℃$$

Answer.

$$T = \frac{m_1r - m_2\lambda}{(m_1 + m_2)c} + \frac{m_1T_1 + m_2T_2}{m_1 + m_2} = 40.08℃$$