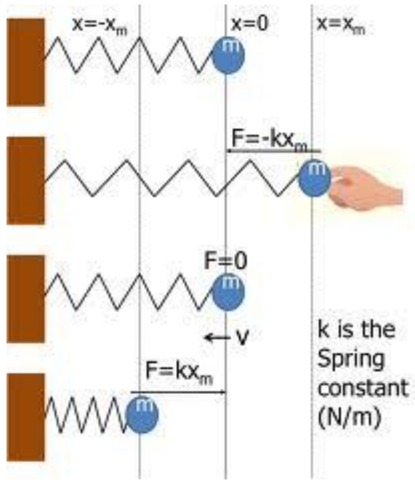
Answer on Question #41966, Physics, Mechanics | Kinematics | Dynamics

Establish the differential equation for a system executing simple harmonic motion (SHM). Show that, for SHM, the velocity and acceleration of the oscillating object is proportional to ω_0 and ω^2_0, respectively, where ω_0 is the natural angular frequency of the object.

Solution:

Simple harmonic motion is typified by the motion of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's Law.



Now since $F = -kx$ is the restoring force and from Newton's law of motion force is give as

$$F = ma,$$

where m is the mass of the particle moving with acceleration a. Thus acceleration of the particle is

$$a = \frac{F}{m} = \frac{-kx}{m}$$

but we know that acceleration $a=dv/dt=d^2x/dt^2$

Thus,

$$\frac{d^2x}{dt^2} = \frac{-kx}{m}$$

This differential equation is known as the simple harmonic equation.

The solution is

$$x = \cos(\omega_0 t + \phi)$$

where A, ω_0 and ϕ are all constants.

We know that velocity of a particle is given by

$$v = \frac{dx}{dt}$$

Now differentiating the displacement of particle x with respect to t

$$v = \frac{dx}{dt} = \omega_0(-\sin(\omega_0 t + \phi))$$

From trigonometry we know that

$$\sin^2 x + \cos^2 x = 1$$

Thus,

$$^2\sin^2(\omega_0 t+\phi) = \ ^2 - \ ^2\cos^2(\omega_0 t+\phi) = \ ^2 - x^2$$

Or

$$\sin(\omega_0 t+\phi) = \sqrt{1-\frac{x^2}{{}^2}}$$

putting this in for velocity we get,

$$v = -A\omega_0\sqrt{1-\frac{x^2}{{}^2}}$$

so it is proportional to ω_0.

Again we know that acceleration of a particle is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}(-A\omega_0\sin(\omega_0 t+\phi)) = -A\omega_0^2\cos(\omega_0 t+\phi) = -\ \omega_0^2 x$$

so it is proportional to $\omega^2{}_0$.