1. A particle of mass m and charge q moves in a circular path in a magnetic field B. Show that its kinetic energy is proportional to r^2 of the radius of curvature of its path. What is the angular momentum of the particle about the center of the circle?

Solution.

The motion of a particle is caused by the Lorentz force: $F = qvB$,

where v is the velocity of the particle.

According the second Newton law, $F = ma$,

where the acceleration during a circular path is $a = \frac{v^2}{r}$, r is the radius of curvature of the path.

So, we can write that $m \cdot \frac{v^2}{r} = qvB$. So, the velocity is $v = \frac{qBr}{m}$.

The kinetic energy of the particle: $E_k = \frac{mv^2}{2} = \frac{m}{2}\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2}{2m}r^2$.

We obtained that the kinetic energy is proportional to r^2.

Let find the angular momentum of the particle about the center of the circle:

$$L = \left|\left[\vec{r} \times \vec{p}\right]\right| = r \cdot mv = r \cdot mv = mr \cdot \frac{qBr}{m} = qBr^2.$$

Answer: $L = qBr^2$.