Answer on Question #41904, Physics, Mechanics | Kinematics | Dynamics

A hot air balloon rising straight up from a level field is tracked by a range finder 500ft from the lift-off point. At the moment the range finder's elevation angle is pi/4, the angle is increasing at the rate of .14rad/min. How fast is the balloon rising at the moment?

Solution

Let h is the height of the balloon, θ is the angle at the range finder to the balloon. So

$$\tan\theta = \frac{h}{500}.$$

Now let's take the derivative thus:

$$\sec^2(\theta)\frac{d\theta}{dt} = \left(\frac{1}{500}\right)\frac{dh}{dt}$$

Solving for $\frac{dh}{dt}$ gives us:

$$\frac{dh}{dt} = 500\sec^2(\theta)\frac{d\theta}{dt}.$$

When $\theta = \frac{\pi}{4}$ and $\frac{d\theta}{dt} = 0.14\frac{rad}{min}$ the rate of change of the height of the balloon is

$$\frac{dh}{dt} = 500ft \cdot \sec^2\left(\frac{\pi}{4}\right)\frac{d\theta}{dt} \cdot 0.14\frac{rad}{min} = 500 \cdot (\sqrt{2})^2 \cdot 0.14 = 140\frac{ft}{min}.$$

Answer: $\mathbf{140\frac{ft}{min}}$.