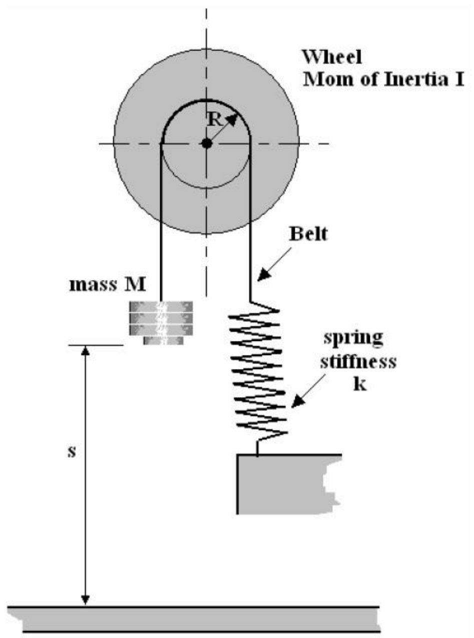
Answer on Question #41669, Physics, Mechanics

An experiment includes a wheel with a moment of inertia (I). A mass (M) is connected to a belt and runs over a drum of radius (R). The other end of the belt is attached to a spring of stiffness (K) that is connected to the ground.

Show that if the mass is pulled down with a force (F) and then released, that the system will oscillate with simple harmonic motion with a frequency given by… F = 1/(2 pi) sqrt(k/(M+i/R^2))

Solution



We suppose the mass is pulled downwards with a force F. This must overcome the inertia of the mass, the inertia of the drum and stretch the spring.

Inertia force to accelerate the drum

The Torque required to overcome inertia of the drum is $T = I\alpha$.

Torque = Force x radius or $T = FR$ and the force is $F = \frac{T}{R}$.

Substitute $T = I\alpha$.

$F_{i1} = \frac{I\alpha}{R}$ where α is the angular acceleration of the drum.

Inertia force to accelerate the drum

$F_{i2} = ma$ where a is the linear acceleration.

Force to stretch the spring

$F_s = kx$ where k is the spring stiffness.

Force balance

$$F = F_{i1} + F_{i2} + F_s = \frac{I\alpha}{R} + ma + kx.$$

The angular acceleration is linked to the linear acceleration by $\alpha = \frac{a}{R}$ where R is the drum radius.

$$F = \frac{I\alpha}{R} + ma + kx = a\left(\frac{I}{R^2} + m\right) + kx.$$

For a free oscillation $F = 0$ hence

$0 = \frac{I\alpha}{R} + ma + kx = a\left(\frac{I}{R^2} + m\right) + kx.$

Make a the subject

$$a = -\left(\frac{k}{\frac{I}{R^2} + m}\right)x.$$

This shows that the acceleration is directly proportional to the displacement so the motion must be simple harmonic. The constant of proportionality is the angular frequency squared so:

$$\omega^2 = \frac{k}{\frac{I}{R^2} + m} \to \omega = \sqrt{\frac{k}{\frac{I}{R^2} + m}} \to f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{I}{R^2} + m}}.$$