

Answer on Question #41632 – Physics – Molecular Physics | Thermodynamics

Question.

a gas is compressed from a volume of 2m^3 to a volume of 1m^3 at a constant pressure of 100N/m^2 . Then it is heated at constant volume by supplying 150J of energy. As a result, the internal energy of the gas increases/decreases by??

$$V_1 = 2 \text{ m}^3$$

$$V_2 = 1 \text{ m}^3$$

$$P = 100 \frac{\text{N}}{\text{m}^2} = \text{const}$$

$$\delta Q_2 = 150 \text{ J}$$

$$dU > 0 \text{ or } dU < 0 ?$$

Solution.

First law of thermodynamics:

$$dU = \delta Q - \delta A$$

dU is an internal energy;

δQ is a heat added to the gas;

δA is a work done by the gas.

Consider two processes in this problem.

$$dU = dU_1 + dU_2$$

In the first process (isobaric compression) the work was made on the gas:

$$\delta A = PdV = P(V_2 - V_1) = -P(V_1 - V_2) < 0 \rightarrow -\delta A = P(V_1 - V_2) > 0$$

$$\delta Q = C_p dT$$

Charles's law (for isobaric process):

$$\frac{V}{T} = \text{const}$$

$$\text{Therefore, in our case } dV < 0 \rightarrow dT < 0 \rightarrow \delta Q = C_p dT < 0$$

$$\text{So, } dU_1 = \delta Q - \delta A = C_p dT + P(V_1 - V_2) > 0$$

$C_p = \frac{i+2}{2}R$, where i is number of degrees of freedom. For example, for monoatomic gas $i = 3$ and for diatomic gas $i = 5$. For n-atomic gas ($n > 3$) $i = 6$.

Ideal gas law: $PV = RT$. For isobaric process: $PdV = RdT$

$$\text{So, } \delta Q = C_p dT = \frac{i+2}{2}RdT = \frac{i+2}{2}PdV = \frac{i+2}{2}P(V_2 - V_1)$$

$$dU_1 = \frac{i+2}{2}P(V_2 - V_1) - P(V_2 - V_1) = \frac{i}{2}P(V_2 - V_1) < 0$$

In the second process gas (isochoric heating) is heated at constant volume. Therefore,

$$dV = 0 \rightarrow \delta A = PdV = 0$$

$$dU_2 = \delta Q_2 > 0$$

So,

$$dU = dU_1 + dU_2 = \frac{i}{2}P(V_2 - V_1) + \delta Q_2$$

Calculate dU :

$$dU = \frac{i}{2}100(1 - 2) + 150 = -50 \cdot i + 150 = -50(i - 3)$$

So, for monoatomic gas ($i = 3$): $dU = 0$.

For diatomic gas ($i = 5$): $dU = -100 \text{ J} < 0$

For n-atomic gas ($i = 6$): $dU = -150 \text{ J} < 0$; $n > 3$

So, dU doesn't increase. It remains constant for monoatomic gas and decreases for n-atomic gas.

Answer.

For monoatomic gas ($i = 3$): $dU = 0$. It's remains constant

For diatomic gas ($i = 5$): $dU = -100 \text{ J} < 0$. It decreases by 100 J

For n-atomic gas ($i = 6$): $dU = -150 \text{ J} < 0$; $n > 3$. It decreases by 150 J