Answer on Question #41354, Physics, Electric Circuits

In L-C oscillation charge in capacitor is maximum at t = 0. The magnetic and electric energy will be equal in magnitude at (T = Time period of oscillation of charge)

Solution

In L-C oscillation charge in capacitor is expressed by the formula

$$q = q_{max}\cos(\omega t + \phi),$$

where q_{max} is the maximal charge in capacitor, ω is the angular frequency, ϕ - the phase constant.

The current in the circuit is

$$I = \frac{dq}{dt} = -\omega q_{max}\sin(\omega t + \phi).$$

To determine the value of the phase angle ϕ, we examine the initial conditions, which in our situation require that at $t = 0, I = 0$ and $q = q_{max}$. Setting $I = 0$ at $t = 0$ in equation for current, we have

$$0 = -\omega q_{max}\sin(\phi)$$

which shows that $\phi = 0$. Therefore, in our case, the expressions for Q and I are

$$q = q_{max}\cos(\omega t),$$

$$I = -\omega q_{max}\sin(\omega t) = -I_{max}\sin(\omega t).$$

The total energy in the circle is

$$U = U_C + U_L = \frac{Cq_{max}^2}{2}\cos^2(\omega t) + \frac{LI_{max}^2}{2}\sin^2(\omega t).$$

The sum $U_C + U_L$ is a constant and equal to the total energy $\frac{Cq_{max}^2}{2} = \frac{LI_{max}^2}{2}$:

$$U = U_C + U_L = \frac{Cq_{max}^2}{2}\cos^2(\omega t) + \frac{Cq_{max}^2}{2}\sin^2(\omega t) = \frac{Cq_{max}^2}{2}(\cos^2(\omega t) + \sin^2(\omega t)) = \frac{Cq_{max}^2}{2}.$$

The magnetic and electric energy will be equal when

$$\cos^2(\omega t) = \sin^2(\omega t) = \frac{1}{2} \rightarrow \omega t = \frac{\pi}{4} + \frac{n\pi}{2}, n = 0,1,2 \ldots$$

We know that $\omega = \frac{2\pi}{T}$, so

$$\left(\frac{2\pi}{T}\right)\cdot t = \frac{\pi}{4} + \frac{n\pi}{2} \rightarrow t = T\left(\frac{1}{8} + \frac{n}{4}\right) = \frac{T}{8} + \frac{nT}{4}, n = 0,1,2 \ldots$$

Answer: $\boldsymbol{\frac{T}{8} + \frac{nT}{4}, n = 0, 1, 2} \ldots$