Answer on Question #41321 - Physics - Molecular Physics

Question.

The ratio of the speed of the electron in the ground state of hydrogen to the speed of light in vacuum is

$\frac{v_e}{c} = ?$

Solution.

The speed of light in vacuum:

$$c = 3 \cdot 10^8 \ \frac{m}{s}$$

The speed of the electron in the ground state of Hydrogen will be found using Bohr's postulates.

Newton's second law of motion for an electron in a circular orbit in the Coulomb force:

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \rightarrow v = \sqrt{\frac{ke^2}{mr}}$$

$m = \ 9.1 \cdot 10^{-31} \ kg$ is the mass of electron;

$v = v_e$ is the velocity of electron;

$k = 9 \cdot 10^9 \ \frac{N \cdot m^2}{C^2}$ is a Coulomb's constant;

$e = 1.6 \cdot 10^{-19} \ C$ is an electron charge;

r is the radius of an orbit(for ground state);

We need to find the radius r and then we find the velocity v.

Use Bohr quantum rule: The angular momentum $L \ = \ mvr$ is an integer multiple of $\hbar$:

$$mvr = n\hbar$$

$$\hbar = 1.05 \cdot 10^{-34} \ J \cdot s$$

Obtain from these two formulas:

$$r_n = \frac{n^2\hbar^2}{kme^2}$$

$$v_n = \frac{ke^2}{n\hbar}$$

$n = 1$ is a ground state of Hydrogen, principal orbit.

So,

$$r_1 = \frac{\hbar^2}{kme^2} = 0.53 \cdot 10^{-11} m$$

$$v_1 = v_e = \frac{ke^2}{\hbar} = 2.2 \cdot 10^6 \ \frac{m}{s}$$

Thus,

$$\frac{v_e}{c} = \frac{2.2 \cdot 10^6}{3 \cdot 10^8} = 0.73 \cdot 10^{-2} = 0.73\%$$

Answer.

$$\frac{v_e}{c} = 0.73 \cdot 10^{-2} = 0.73\%$$