Answer on Question #41254 – Physics - Molecular Physics

Question.

Two gases have same initial pressure, volume and temperature. They expand to the same final volume, one adiabatically and the other isothermally

(1) The final pressure is greater for isothermal process
(2) The final temperature is greater for the isothermal process
(3) The work done by the gas is greater for the isothermal process
(4) All of these

P_1 is an initial pressure
V_1 is an initial volume
T_1 is an initial temperature
P_{end} is the final pressure
T_{end} is an initial temperature

Solution.

By definition work done is:

$$\delta A = PdV$$

$$A = \int \delta A = \int PdV$$

Adiabatic process:

$$TV^{\gamma-1} = const$$

$$T_1 V_1^{\gamma-1} = TV^{\gamma-1} \rightarrow T_{end} = T_1 \left(\frac{V_1}{V}\right)^{\gamma-1}$$

$$PV^{\gamma} = const$$

$$P_1 V_1^{\gamma} = PV^{\gamma} \rightarrow P_{end} = P_1 \left(\frac{V_1}{V}\right)^{\gamma}$$

$$A = \int \delta A = \int PdV = \int_{V_1}^{V_2} P_1 V_1^{\gamma} \frac{dV}{V^{\gamma}} = \gamma P_1 V_1^{\gamma} \left(\frac{1}{V_1^{\gamma-1}} - \frac{1}{V_2^{\gamma-1}}\right) = \gamma P_1 V_1 \left(1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right)$$

Isothermal process:

$$T_{end} = T_1$$

$$PV = RT_1$$

$$P = \frac{RT_1}{V} \rightarrow P_{end} = \frac{P_1 V_1}{V}$$

$$A = \int \delta A = \int PdV = \int_{V_1}^{V_2} RT_1 \frac{dV}{V} = RT_1 ln\left(\frac{V_2}{V_1}\right) = P_1 V_1 ln\left(\frac{V_2}{V_1}\right)$$

So,

$$P_{end}^{isotherm} = \frac{P_1 V_1}{V} > P_{end}^{adiabatic} = P_1 \left(\frac{V_1}{V}\right)^{\gamma} \leftrightarrow P_{end}^{isotherm} > P_{end}^{adiabatic}$$

$$T_{end}^{isotherm} = T_1 > T_{end}^{adiabatic} = T_1 \left(\frac{V_1}{V}\right)^{\gamma - 1} \leftrightarrow T_{end}^{isotherm} > T_{end}^{adiabatic}$$

$A = \int PdV$, therefore the work done by the gas is greater for process with greater final pressure and temperature → The work done by the gas is greater for the isothermal process.

You can also see it from:

$$A^{isotherm} = P_1 V_1 ln\left(\frac{V_2}{V_1}\right) > A^{adiabatic} = \gamma P_1 V_1 \left(1 - \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right) \leftrightarrow A^{isotherm} > A^{adiabatic}$$

Thus, the final pressure, temperature and work done by the gas are greater for isothermal process than for adiabatic process.

Answer.

(4) All of these