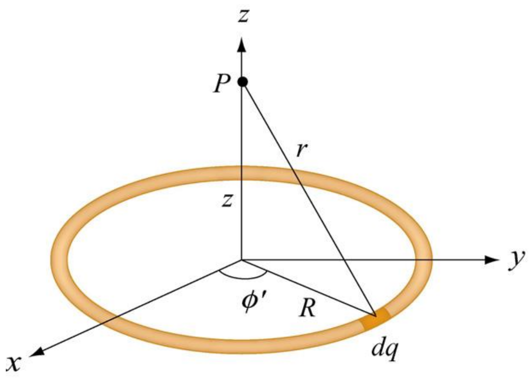
Answer on Question #41236, Physics, Electrodynamics

The electric potential due to a uniformly charged ring of radius R having charge Q at a distance $\sqrt{3}R$ on its axis.

Solution

Consider a uniformly charged ring of radius R and charge density $\lambda = \frac{Q}{2\pi R}$.



Consider a small differential element $dl = Rd\phi'$ on the ring. The element carries a charge

$$dq = \lambda dl = \lambda R d\phi',$$

and its contribution to the electric potential at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\phi'}{\sqrt{R^2+z^2}},$$

where $r = \sqrt{R^2+z^2}$.

The electric potential at P due to the entire ring is

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{\sqrt{R^2+z^2}}\oint d\phi' = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\lambda R}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}},$$

where we have substituted $Q = 2\pi\lambda R$ for the total charge on the ring.

In our case $z = \sqrt{3}R$ electric potential is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+(\sqrt{3}R)^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+3R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} = \frac{Q}{8\pi\varepsilon_0 R}.$$

Answer: $\frac{Q}{8\pi\varepsilon_0 R}$.