Answer on Question #41179, Physics, Mechanics | Kinematics | Dynamics

A simple harmonic oscillator has amplitude A angular velocity ω and mass m. Then average energy in one time period will be

Solution:

Linear Harmonic oscillator.

Mechanical model: mass m on a spring characterized by a spring constant k.

Elastic restoring force $F = -kx$ is balanced according to Newton's second law

$$F = ma$$

$$m\ddot{x} = -kx$$

The equation of motion

$$\ddot{x} + \omega^2 x = 0$$

where

$$\omega = \sqrt{\frac{k}{m}}$$

Such system oscillates with amplitude A and angular frequency ω.

Consider solution

$$x = A\sin(\omega t + \varphi)$$

The velocity v is equal to

$$v = \dot{x} = A\omega\cos(\omega t + \varphi)$$

and thus the kinetic energy

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \varphi) = K_0\cos^2(\omega t + \varphi)$$

where the maximum kinetic energy is equal to

$$K_0 = \frac{1}{2}mA^2\omega^2 = \frac{1}{2}kA^2$$

The potential energy – work done by applied force displacing the system from 0 to x

$$U(x) = \int_0^x kx dx = \frac{1}{2}kx^2$$

Substituting x

$$U(x) = \frac{1}{2}kA^2\sin^2(\omega t + \varphi) = U_0\sin^2(\omega t + \varphi)$$

where U_0 is the maximum potential energy (for x=A)

$$U_0 = \frac{1}{2}kA^2$$

The average values over one oscillation period are calculated using the definition

$$\langle f\rangle = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f(t)dt$$

Thus:

$$\langle U\rangle = \frac{\int_0^T U dt}{\int_0^T dt} = \frac{\int_0^T U_0 \sin^2(\omega t + \varphi)\, dt}{T} = \frac{1}{2}U_0 = \frac{1}{2}kA^2$$

and

$$\langle K\rangle = \frac{\int_0^T K dt}{\int_0^T dt} = \frac{1}{2}K_0 = \frac{1}{2}kA^2$$

The sum of the kinetic and potential energies in a simple harmonic oscillator is a constant, i.e., KE+PE=constant. The energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates.

Thus,

$$\langle E\rangle = \langle K\rangle + \langle U\rangle = \frac{1}{2}kA^2 + \frac{1}{2}kA^2 = kA^2 = mA^2\omega^2$$

Answer. $\langle E\rangle = mA^2\omega^2$.