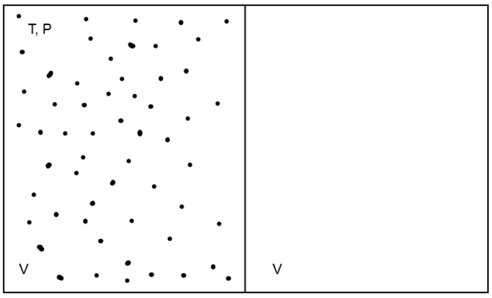
Answer on Question #41155 - Physics - Molecular Physics | Thermodynamics

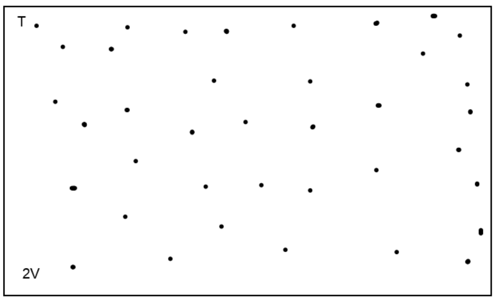
Question.

Two identical containers are connected by very narrow tube. If one container is filled with ideal gas at temp T and pressure P and other one is evacuated. When tube connecting is opened then temperature of gas does not change. Why?

Proof.



Start



End

In this process energy doesn't change. So,

$$E_{start} = E_{end} \to dU = 0$$

First law of thermodynamics:

$$dU = TdS - PdV$$

In our case:

$$dU = TdS - PdV = 0 \quad \textbf{(1)}$$

Entropy S is the function of temperature and volume:

$$S = S(T, V)$$

Therefore:

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

Substitute in equation **(1)**:

$$dU = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV - PdV$$

$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$ - heat capacity

By definition the differential of enthalpy is:

$$dH = TdS + VdP$$

Using the properties of the first and second derivatives we obtain:

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

Substitute in equation for dU:

$$dU = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV - PdV = C_V dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] dV$$

$$dU = 0 \rightarrow C_V dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] dV = 0$$

And finally we get the temperature change with a change in volume:

$$\left(\frac{\partial T}{\partial V}\right)_E = \frac{1}{C_V}\left[P - T\left(\frac{\partial P}{\partial T}\right)_V\right]$$

All this we have done for any gas. But now let us remember that we have the ideal gas.

Ideal gas law (for 1 mol):

$$PV = RT \rightarrow P = \frac{RT}{V}$$

Therefore:

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V}$$

Substitute this in our final equation:

$$\left(\frac{\partial T}{\partial V}\right)_E = \frac{1}{C_V}\left[P - T\left(\frac{\partial P}{\partial T}\right)_V\right] = \frac{1}{C_V}\left[P - T\frac{R}{V}\right] = \frac{1}{C_V}[P - P] = 0$$

Thus, $dT = 0$, i.e. temperature of gas doesn't change.

Notation. But this is true only for ideal gas. Common formula is:

$$\left(\frac{\partial T}{\partial V}\right)_E = \frac{1}{C_V}\left[P - T\left(\frac{\partial P}{\partial T}\right)_V\right]$$