

Answer on Question #41145, Physics, Mechanics | Kinematics | Dynamics

A stone hangs from the free end of a sonometer wire whose vibrating length when tuned to a tuning fork is 40 cm. When the stone hangs wholly immersed in water, the resonant length is reduced to 30 cm. The relative density of the stone is

Solution:

The frequency of a string of length L , mass per unit length m and stretched with a tension T is given by

$$f = \frac{p}{2L} \sqrt{\frac{T}{m}}$$

where p is the number of segments in which the string is vibrating.

In the fundamental mode (the first harmonic), the string vibrates in one segment ($p = 1$).

Given:

$$L_1 = 0.4 \text{ m,}$$

$$L_2 = 0.3 \text{ m,}$$

$$RD = ?$$

In our case,

$$f = \frac{p}{2L_1} \sqrt{\frac{T_1}{m}} = \frac{p}{2L_2} \sqrt{\frac{T_2}{m}}$$

Thus,

$$\sqrt{\frac{T_1}{T_2}} = \frac{L_1}{L_2}$$
$$\frac{T_1}{T_2} = \left(\frac{L_1}{L_2}\right)^2 = \left(\frac{0.4}{0.3}\right)^2 = \frac{16}{9}$$

The tension is equal to weight

Relative density (with respect to water) can then be calculated using the following formula:

$$RD = \frac{W_{air}}{W_{air} - W_{water}}$$

where W_{air} is the weight of the sample in air (measured in newtons),

W_{water} is the weight of the sample in water (measured in the same units).

$$RD = \frac{W_1}{W_1 - W_2} = \frac{T_1}{T_1 - T_2}$$
$$RD = \frac{1}{1 - \frac{T_2}{T_1}} = \frac{1}{1 - \frac{9}{16}} = \frac{1}{\frac{7}{16}} = \frac{16}{7} = 2.29 = 2.3$$

Answer. $RD = \frac{16}{7} = 2.3.$