Answer on Question #41138 – Physics – Mechanics | Kinematics | Dynamics

A body is released from a height equal to the radius R of the earth . The velocity of the body when it strikes the surface of the earth will be

Solution:
Since the initial velocity of the body is zero, it's total energy is:

$$\mathrm{E} = -\frac{\mathrm{GmM}}{\mathrm{r}} \qquad (1)$$

where m is mass of the body, M is the mass of the earth and r is distance from the centre of the earth. When the body reaches the earth, let its velocity be v and its distance from the centre of the earth is the earth's radius R. Therefor, the energy now is

$$\mathrm{E} = \frac{1}{2}\mathrm{mv}^2 - \frac{\mathrm{GmM}}{\mathrm{R}} \qquad (1)$$

Equating $(1)\mathrm{and}(2)$ we get

$$\frac{1}{2}\mathrm{mv}^2 - \frac{\mathrm{GmM}}{\mathrm{R}} = -\frac{\mathrm{GmM}}{\mathrm{r}}$$

$$\mathrm{v}^2 = 2\mathrm{GM}\left(\frac{1}{\mathrm{R}} - \frac{1}{\mathrm{r}}\right)$$

Also $\mathrm{g} = \frac{\mathrm{GM}}{\mathrm{R}^2}$.Therefore $\mathrm{GM} = \mathrm{gR}^2$. Using this in above equation we get

$$\mathrm{v} = \mathrm{R}\left(2\mathrm{g}\left(\frac{1}{\mathrm{R}} - \frac{1}{\mathrm{r}}\right)\right)^{\frac{1}{2}}$$

Now $\mathrm{r} = 2\mathrm{R}\,(\mathrm{given})$. Therefore

$$\mathrm{v} = \mathrm{R}\left(2\mathrm{g}\left(\frac{1}{\mathrm{R}} - \frac{1}{\mathrm{r}}\right)\right)^{\frac{1}{2}} = \sqrt{\mathrm{gR}}$$

Answer: velocity of the body when it strikes the surface of the earth will be $\sqrt{\mathrm{gR}}$.