Answer on Question #41081, Physics, Molecular Physics | Thermodynamics

What is transport phenomenon? How does it arise? Derive an expression for the coefficient of viscosity of a gas. Discuss its temperature and pressure dependence.

Answer

What is transport phenomenon?

In physics, transport phenomena are all irreversible processes of statistical nature stemming from the random continuous motion of molecules.

Every aspect of transport phenomena is grounded in two primary concepts: the conservation laws, and the constitutive equations.

The conservation laws, which in the context of transport phenomena are formulated as continuity equations, describe how the quantity being studied must be conserved within the universe of the question. The constitutive equations describe how the quantity in question responds to various stimuli via transport. Prominent examples include Fourier's Law of Heat Conduction and the Navier-Stokes equations, which describe, respectively, the response of heat flux to temperature gradients and the relationship between fluid flux and the forces applied to the fluid. These equations also demonstrate the deep connection between transport phenomena and thermodynamics, a connection that explains why transport phenomena are irreversible. Almost all of these physical phenomena ultimately involve systems seeking their lowest energy state in keeping with the principle of minimum energy. As they approach this state, they tend to achieve true thermodynamic equilibrium, at which point there are no longer any driving forces in the system and transport ceases. The various aspects of such equilibrium are directly connected to a specific transport: heat transfer is the system's attempt to achieve thermal equilibrium with its environment, just as mass and momentum transport move the system towards chemical and mechanical equilibrium.

Examples of transport processes include heat conduction (energy transfer), fluid flow (momentum transfer), molecular diffusion (mass transfer), radiation and electric charge transfer in semiconductors.

How does it arise?

Transport phenomena can be caused by the action of an external electric field or by the presence of spatial inhomogeneities of composition, temperature, or average velocity of the particles of the system. The transport of the physical entity occurs in the direction opposite to the gradient of the entity. Transport phenomena bring a system closer to the equilibrium state.

Transport phenomena also occur in heterogeneous systems consisting of homogeneous parts, or subsystems, separated by natural interfaces such as a liquid and its vapor.

Derive an expression for the coefficient of viscosity of a gas.

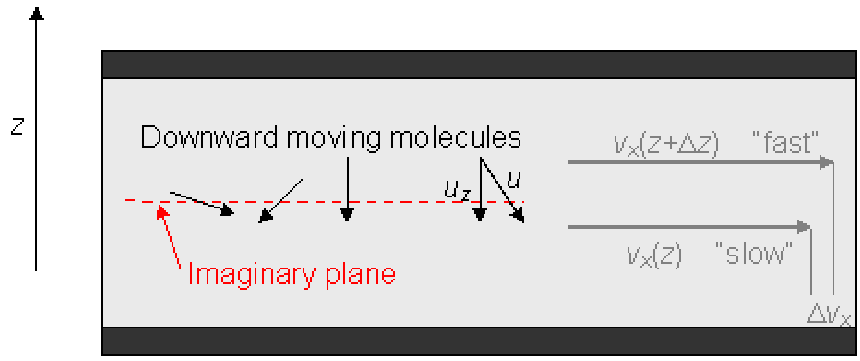
The way to find the viscosity of a gas is to calculate the rate of z-direction (downward) transfer of x-momentum.

The moving top plate maintains a steady horizontal flow pattern, the x-direction speed at height z $v(z) = \frac{v_0 z}{d}$.

The moving plate is feeding x-direction momentum into the gas at a rate $\frac{F}{A} = \frac{\eta v_0}{d}$, this momentum moves down through the gas at a steady rate, and the coefficient of viscosity tells us what this rate of momentum flow is for a given velocity gradient.

We'll make the simplifying assumption that the molecules all have speed u, and travel a distance l between collisions. We take the density of molecules to be n, the molecular mass m. To begin thinking about x-direction momentum moving downwards, imagine some plane parallel to the plates and between them. Molecules from above are shooting through this plane and colliding with molecules in the slower moving gas below, on average transferring a little extra momentum in the x-direction to the slower stream. At the same time, some molecules from the slower stream are shooting upwards and will slow down the faster stream.

Let's consider first the molecules passing through the imaginary plane from above: we're only interested in the molecules already moving downwards, that's half of them, so a molecular density of $\frac{n}{2}$. If we assume for simplicity that all the molecules move at the same speed u, then the average downward speed of these molecules $\overline{u_z} = \frac{u}{2}$.



Thus the number of molecules per second passing through the plane from above is $\frac{n}{2}|\overline{u_z}| = \frac{nu}{4}$ and the same number are of course coming up from below.

Let us assume molecule travels a distance l from its last collision in the "fast" stream to its first in the "slow" stream. The average distance between collisions is called the mean free path, here "mean" is used in the sense of "average", and is denoted by l. The distance traveled in the downward direction is $\Delta z = \frac{lu_z}{z}$, so the (x-direction) speed difference between the two streams is

$$\Delta v = \frac{dv(z)}{dz}\Delta \mathrm{z} = \frac{\mathrm{v}_0}{\mathrm{d}}\frac{lu_z}{z}.$$

The molecule has mass m, so on average the momentum transferred from the fast stream to the slow stream is $\Delta p = m\Delta v$. With our simplifying assumption that all molecules have the same speed u, all downward values of u_z between 0 and u are equally likely, and the density of downward-moving molecules is $\frac{n}{2}$, so the rate of transfer of momentum by downward-moving molecules through the plane is

$$\frac{n}{2}\overline{u_z m\Delta v} = \frac{nm}{2}\cdot l\cdot\frac{\overline{u_z^2}}{u}\cdot\frac{v_0}{d}.$$

At the same time, molecules are moving upwards form the slower streams into the faster ones, and the calculation is exactly the same. These two processes have the same sign: in the first case, the slower stream is gaining forward momentum from the faster, in the second, the faster stream is losing forward momentum, and in both cases total forward momentum is conserved. Therefore, the two processes make the same contribution, and the total momentum flow rate (per unit area) across the plane is

$$\text{momentum transfer rate} = nm \cdot l \cdot \frac{\overline{u_z^2}}{u} \cdot \frac{v_0}{d} = \frac{1}{3} nmul \frac{v_0}{d}$$

using $\overline{u_z^2} = \frac{u^2}{3}$.

Evidently this rate of downward transfer of x-direction momentum doesn't depend on what level between the plates we choose for our imaginary plane, it's the same momentum flow all the way from the top plate to the bottom plate: so it's simply the rate at which the moving top plate is supplying x-direction momentum to the fluid,

$$\text{momentum suply rate} = \frac{\mathrm{F}}{\mathrm{A}} = \frac{\eta v_0}{d}.$$

Since the momentum supplied moves steadily downwards through the fluid, the supply rate is the transfer rate, the two equations above are for the same thing, and we deduce that the coefficient of viscosity

$$\eta = \frac{1}{3} nmul.$$

Discuss its temperature and pressure dependence.

It's easy to see one reason why <u>the viscosity increases with temperature</u>: from $\eta = \frac{1}{3} nmul$, η is proportional to the average molecular speed u, and since this depends on temperature as $\frac{1}{2} m\overline{u^2} = \frac{3}{2} kT$, this factor contributes a $\sqrt{T}$ dependence.

$$l \approx \frac{1}{\sqrt{2} n \sigma_0},$$

where σ_0 is scattering cross-section.

Thus factor n cancels in $\eta = \frac{1}{3} nmul$, and one obtains

$$\eta = \frac{1}{3\sqrt{2}} \frac{m}{\sigma_0} u.$$

But the mean molecular speed depends only on temperature but not on the gas density n, or equivalently, on the gas pressure $\bar{p} = nkT$. So <u>the viscosity don't depend on the gas pressure.</u>