Answer on Question #40982, Physics, Mechanics | Kinematics | Dynamics

Two waves 1 and 2 are present on a string:

y1 = (35 mm) Sin[(8.4m^-1)x - (15.7s^-1)t]

y2 = (35 mm) Sin[(8.4m^-1)x+ (15.7s^-1)t]

(i) Write the expression for the resultant wave, y = y1 + y2 in the form of wave function for a standing wave. (ii) Determine the x coordinates of the first two antinodes, starting at the origin and progressing towards + x direction. (iii) Determine the x coordinate of the node that is between the antinodes of part (ii).

Solution

(i) Two waves 1 and 2 are present on a string:

$$y_1 = 35\sin(8.4x - 15.7\text{t}),$$

$$y_2 = 35\sin(8.4x + 15.7\text{t}).$$

The sum of these two waves is:

$$y = y_1 + y_2 = 35\sin(8.4x - 15.7\text{t}) + 35\sin(8.4x + 15.7\text{t}).$$

We can use formulae for the sum of sinuses:

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}.$$

So

$$y = y_1 + y_2 = 70\sin(8.4x)\cos(15.7\text{t}).$$

(ii) The positions of the antinodes are given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots, \frac{(2n-1)\lambda}{4} \quad n = 1,2,3 \dots$$

In our case $k = \frac{2\pi}{\lambda} = 8.4\frac{rad}{m}$ and the wavelength is $\lambda = \frac{2\pi}{8.4} = 0.748\ m$.

The x coordinates of the first two antinodes are

$$x_1 = \frac{\lambda}{4} = 0.187\ m,$$

$$x_2 = \frac{3\lambda}{4} = 0.561\ m.$$

(iii) The node is located at

$$x = \frac{\lambda}{2} = 0.374m.$$

Answer: (i) $70\sin(8.4x)\cos(15.7\text{t})$; (ii) $0.187\ m, 0.561\ m$; (iii) $0.374m$.