

A boy standing still throws a ball at an angle  $60^\circ$ , with the horizontal as shown in the figure below. At the moment the boy releases the ball, he ran eastwards to catch the ball. If the boy caught the ball in exactly 2 seconds at the same height from which he threw it. Neglect the effect of air resistance or wind on the ball.

- What is the magnitude of the initial velocity,  $|v_0|$ , with which the boy threw the ball?
- With what average velocity (magnitude and direction) must the boy run in order to catch ball at the point shown?
- What is the speed of the ball when the boy catches it?

### Solution

The initial velocity of the ball in vertical direction is  $v_{0y} = v_0 \sin 60^\circ$ . The height of the ball after time  $t$  is

$$h = v_{0y}t - \frac{gt^2}{2}.$$

The height of the ball after 2 seconds is

$$0 = v_{0y} \cdot 2 - \frac{9.8 \cdot 2^2}{2} \rightarrow v_{0y} = 9.8 \frac{m}{s}.$$

- The magnitude of the initial velocity is

$$v_0 = \frac{v_{0y}}{\sin 60^\circ} = \frac{9.8 \frac{m}{s}}{\frac{\sqrt{3}}{2}} = 11.3 \frac{m}{s}.$$

- We already know that the direction of the boy is eastwards.

The average velocity of the boy in order to catch ball is equal velocity of the ball in horizontal direction:

$$v_{\text{boy}} = v_x = v_{0x} = v_0 \cos 60^\circ = v_0 \cdot \frac{1}{2} = \frac{11.3 \frac{m}{s}}{2} = 5.65 \frac{m}{s}.$$

- The speed of the ball when the boy catches it is

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2},$$

where  $v_{fx} = v_{0x} = 5.65 \frac{m}{s}$  and  $v_{fy} = v_{0y} - gt = 9.8 - 2 \cdot 9.8 = -9.8 \frac{m}{s}$ .

So

$$v_f = \sqrt{v_{0x}^2 + (-v_{0y})^2} = \sqrt{v_{0x}^2 + v_{0y}^2} = |v_0| = 11.3 \frac{m}{s}.$$

