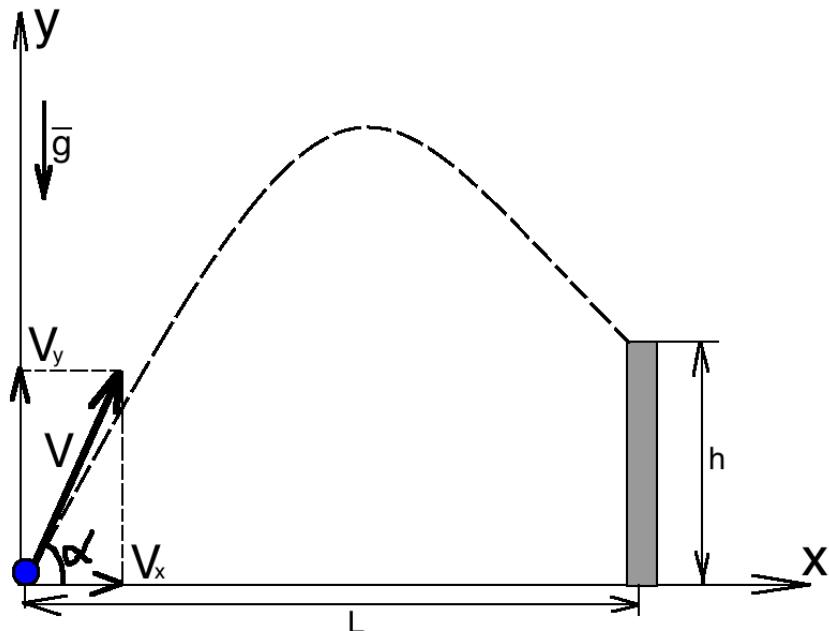


Answer on Question #40952 – Physics - Mechanics | Kinematics | Dynamics

A boy kicks his football (soccer ball) with an initial speed of 30 m/s at an angle 45° above the horizontal aiming to hit a 20 m-tall post which is 25 m away from him. Neglecting air resistance, find: a) with what velocity will he hit the post, b) where exactly (x, y) will he hit it?



Solution:

$L = 25 \text{ m}$ – horizontal range of the ball;

$V = 30 \frac{\text{m}}{\text{s}}$ – speed of the soccer ball;

$\alpha = 45^\circ$ – the angle of projection;

$H = 20 \text{ m}$ – height of the post;

h – height at which the ball will hit the post;

Equation of the motion for the ball, directed at angle α : (t – time of the flight)

$$V_x = V \cos \alpha; V_y = V \sin \alpha;$$

$$x: L = Vt \cos \alpha$$

$$t = \frac{L}{V \cos \alpha} \quad (1)$$

$$y: h = Vt \sin \alpha - \frac{gt^2}{2} \quad (2)$$

(2)in(1)

$$\begin{aligned} h &= \frac{L}{V \cos \alpha} \cdot V \sin \alpha - \frac{g \left(\frac{L}{V \cos \alpha} \right)^2}{2} = L \tan \alpha - \frac{gL^2}{2V^2 \cos^2 \alpha} = \\ &= 25 \text{ m} \cdot \tan 45^\circ - \frac{9.8 \frac{\text{m}}{\text{s}^2} \cdot (25 \text{ m})^2}{2 \cdot \left(30 \frac{\text{m}}{\text{s}} \right)^2 \cdot \cos 45^\circ} = 18.2 \text{ m} \end{aligned}$$

Ball hit the post, so it traveled distance L along the X-axis and distance h along Y-axis, hence, ball will hit the post in point $(L, h) = (25m, 18.2m)$

Principle of conservation of energy: initial energy of the ball is equal to the energy when the ball hit the post:

$$W_1 = W_2 \quad (3)$$

Initial energy of the ball (initial level on the ground, thus $PE = 0$)

$$W_1 = W_{KE1} = \frac{mV^2}{2} \quad (4)$$

Moment of hitting the post:

$$W_2 = W_{KE2} + W_{PE2} = \frac{mV_1^2}{2} + mgh \quad (5)$$

(5) and (4) in (3):

$$\frac{mV^2}{2} = \frac{mV_1^2}{2} + mgh$$

$$V_1^2 = V^2 - 2gh$$

$$V_1 = \sqrt{V^2 - 2gh} = \sqrt{\left(30 \frac{m}{s}\right)^2 - 2 \cdot 9.8 \frac{N}{kg} \cdot 18.2m} = 23.3 \frac{m}{s}$$

Answer: a) ball will hit the post with velocity $23.3 \frac{m}{s}$

b) point of the hit: $(25m, 18.2m)$.