Answer on Question#40854 – Physics – Mechanics

A 2000 kg satellite orbits the earth at a height of 300 km. What is the speed of the satellite and its period? Take G=6:67×10−11Nm2=kg2, Mass of the earth is 5:98×1024kg

Solution:

$M = 5.98 \times 10^{24}\text{kg} - \text{mass of Earth}$

$G = 6.67 \times 10^{-11}\ \text{N} \cdot \dfrac{\text{m}^2}{\text{kg}^2} - \text{gravitational constant;}$

$R = 6.3 \times 10^{6}\text{m} - \text{radius of Earth;}$

$h = 300\text{km} - \text{satellite orbits heigth above the Earth;}$

Consider a satellite with mass m orbiting a central body with a mass of mass M_{earth}. If the satellite moves in circular motion, then the net centripetal force acting upon this orbiting satellite is given by the relationship

$$F_{net} = m\frac{v^2}{R} \quad (1)$$

This net centripetal force is the result of the gravitational force that attracts the satellite towards the central body and can be represented as

$$F_{grav} = G\frac{mM}{R^2} \quad (2)$$

Since F_{net} = F_{grav}, the above expressions for centripetal force and gravitational force can be set equal to each other. Thus,

$$(1) = (2):$$

$$m\frac{v^2}{R} = G\frac{mM}{R^2}$$

Observe that the mass of the satellite is present on both sides of the equation; thus it can be canceled by dividing through by m. Then both sides of the equation can be multiplied by R, leaving the following equation.

$$v^2 = \frac{GM}{R}$$

$$v = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67 \times 10^{-11}\ \text{N} \cdot \frac{\text{m}^2}{\text{kg}^2} \cdot 5.98 \times 10^{24}\text{kg}}{6.3\ \times 10^{6}\text{m} + 300 \times 10^{3}\text{m}}} = 7774\frac{\text{m}}{\text{s}}$$

The equation that is useful in describing the motion of satellites is Newton's form of Kepler's third law. The period of a satellite (T) and the mean distance from the central body $(R + h)$ are related by the following equation:

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM}$$

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2 \cdot 3.14\sqrt{\frac{(6.3\ \times 10^{6}\text{m} + 300 \times 10^{3}\text{m})^3}{6.67 \times 10^{-11}\ \text{N} \cdot \frac{\text{m}^2}{\text{kg}^2} \cdot 5.98 \times 10^{24}\text{kg}}} = 5332\text{s}$$

$$= 1.48\ \text{hours}$$

Answer: speed of the satellite: $v = 7774\frac{\text{m}}{\text{s}}$;

period of the satellite: $\mathrm{T} = 1.48\ \mathrm{hours}$.