Answer on Question #40847, Physics, Molecular Physics

Calculate the work done by one mole of a van der Waals gas if during its isothermal expansion its volume increases from 1 m^3 to 2 m^3 at a temperature 300 K. Take a=1.39×10^{-6} atm m^6 mol^{-2} and b= 39.1×$10^{-6}m^3$ mol^{-1}.

Solution:

We can start from the definition of work,

$$W = -\int PdV$$

An expression for P is required – this comes from the van der Waals equation. Solving the van der Waals equation for P leads to

$$P = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

The equation uses the following state variables: the pressure P, total volume V, number of moles n, and absolute temperature of the system T.

Substituting this relation into the equation for work yields

$$W = -\int_{V_1}^{V_2} \left(\frac{nRT}{V - nb} - \frac{an^2}{V^2}\right) dV$$

Since the process is isothermal, T can be pulled out of the integral along with the other constants to give

$$W = -nRT \int_{V_1}^{V_2} \frac{1}{V - nb} dV + an^2 \int_{V_1}^{V_2} \frac{1}{V^2} dV$$

Integrating leads to the result

$$W = -nRTln\left(\frac{V_2 - nb}{V_1 - nb}\right) - an^2\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

1 atm = 101325 Pa

a =1.39×10^{-6} atm m^6 mol^{-2}=1.39·101325·10^{-6} = 0.1408 Pa· m^6· mol^{-2}

So,

$$W = -8.31 \cdot 300 \cdot ln\left(\frac{2 - 39.1 \cdot 10^{-6}}{1 - 39.1 \cdot 10^{-6}}\right) - 0.1408 \cdot \left(\frac{1}{2} - \frac{1}{1}\right) = -1727.99 \text{ J}$$

Answer. $W = -1728$ J.