Answer on Question #40824 – Physics – Mechanics

A tunnel is dug across diameter of earth.A ball is released from surface of earth into the tunnel. The velocity of ball when it is at a distance R/2 from centre of earth is answer in terms of G,M and R
R is radii of earth and M is mass of earth

Solution:
Gravitational potential at a point on the surface of the Earth

$$U_1 = -\frac{GM}{R} \quad (1)$$

If Earth is assumed to be a solid sphere, then the gravitational potential at the distance $x = \frac{R}{2}$ from the centre of Earth is

$$U_2 = -\int_{\infty}^{x} \vec{g} \cdot \overrightarrow{dx} = -\left[\int_{\infty}^{R} \vec{g}_{outside} \cdot \overrightarrow{dx} + \int_{R}^{x} \vec{g}_{inside} \cdot \overrightarrow{dx}\right] =$$

$$= \int_{\infty}^{R} \frac{GM}{x^2} dx + \int_{R}^{x} \frac{GMx}{R^3} dx = -\frac{GM}{R} + \frac{GM}{2R^3}(x^2 - R^2) =$$

$$= -\frac{GM}{2R^3}(3R^2 - x^2)$$

$$x = \frac{R}{2} \Longrightarrow$$

$$U_2 = -\frac{GM}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right) = -\frac{GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = -\frac{11GMR^2}{8R^3} = -\frac{11GM}{8R} \quad (2)$$

Loss in the potential energy:

$$m(U_1 - U_2) = m\left(-\frac{GM}{R} + \frac{11GM}{8R}\right) = \frac{3}{8}\frac{GMm}{R}$$

Now, gain in the kinetic energy = loss in potential energy:

$$\frac{1}{2}mv^2 = \frac{3}{8}\frac{GMm}{R}$$

$$v^2 = \frac{3}{4}\frac{GM}{R}$$

$$v = \sqrt{\frac{3GM}{4R}}$$

Answer: The velocity of ball when it is at a distance R/2 from centre of earth is $v = \sqrt{\frac{3GM}{4R}}$.