Answer on Question #40800, Physics, Mechanics

What is the orbital radius and speed of a synchronous satellite which orbits the earth once every 24h? Take G=6.67·10^{-11} Nm^2/kg^2, Mass of the Earth is 5.98·10^{24} kg.

Solution:

If we just consider the earth-satellite system, then there is only one force acting on the satellite. Suppose the mass of the satellite is m, the mass of the earth is M, and the radius of the satellite's orbit is R.

$$F = G\frac{mM}{R^2} = ma$$

The gravitational force on the satellite provides a centripetal acceleration that pulls the satellite inward, holding it in a circular orbit. A generic formula for the magnitude of the centripetal acceleration is a=Rω^2, where omega is the angular frequency of the satellite's orbit.

The equation relating the angular velocity omega and the time period T is

$$\omega = \frac{2\pi}{T}$$

Thus,

$$G\frac{M}{R^2} = R\left(\frac{2\pi}{T}\right)^2$$

Rearrange and obtain

$$R^3 = \frac{GMT^2}{4\pi^2}$$

Thus,

$$R = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$T = 24\text{ h} = 24 \cdot 3600 = 86400\text{ s}.$

So,

$$R = \sqrt[3]{\frac{6.67 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24} \cdot 86400^2}{4 \cdot 3.14^2}} = 4.23 \cdot 10^7\text{ m}$$

The satellite travels around the entire circumference of the circle — which is

$$L = 2\pi R$$

if R is the radius of the orbit — in the period, T.

This means the orbital speed must be

$$v = \frac{L}{T} = \frac{2\pi R}{T} = \frac{2 \cdot 3.14 \cdot 4.23 \cdot 10^7}{86400} = 3074.6\text{ m/s}$$

Answer. $R = 4.23 \cdot 10^7\text{ m}$, $v = 3074.6\text{ m/s}$.