Answer on Question #40791, Physics, Solid State Physics

Calculate the fermi energy and the corresponding fermi temperature of aluminium (trivalent metal) using the following data: M_{al}= 0.02698 kg/mol and ρ = 2670 kg/m^3 .

Solution:

The Fermi energy is the maximum energy occupied by an electron at 0K. By the Pauli exclusion principle, we know that the electrons will fill all available energy levels, and the top of that "Fermi sea" of electrons is called the Fermi energy or Fermi level.

We can express the Fermi energy in terms of the free electron density.

$$E_F = \left(\frac{\hbar^2}{2m}\right)(3\pi^2 n)^{2/3}$$

where

the Planck constant is $h = 6.63 \cdot 10^{-34}\ \mathrm{J \cdot s} = 4.136 \cdot 10^{-15}\ \mathrm{eV \cdot s}$,

$\hbar = \frac{h}{2\pi} = 1.0546\ \cdot 10^{-34}\ \mathrm{J \cdot s}$,

the electron mass is $m = 9.11 \cdot 10^{-31}\ \mathrm{kg}$,

the free electron density is

$$n = \frac{\text{Number of free electrons}}{\text{Volume}} = \frac{N}{V}$$

The amount of substance in a aluminium sample can be calculated as m/ M_{al}, where m is the mass and M_{al} = 26.98 g/mol is the atomic mass.

Thus, the number of free electrons is

$$N = \frac{m}{M_{al}} \cdot 3 \cdot N_A$$

where the Avogadro constant is $N_A = 6.022 \cdot 10^{23}\ \mathrm{mol^{-1}}$, multiplier 3 is number of electrons per atom.

The Volume is

$$V = \frac{m}{\rho}$$

Thus,

$$n = \frac{N}{V} = \frac{m}{M_{al}} \cdot 3N_A \cdot \frac{\rho}{m} = \frac{3N_A\rho}{M_{al}}$$

$$n = \frac{N_A\rho}{M_{al}} = \frac{3 \cdot 6.022 \cdot 10^{23} \cdot 2670}{0.02698} = 1.788 \cdot 10^{29}\ \mathrm{electrons/m^3}$$

Thus, the Fermi energy is

$$E_F = \left(\frac{(1.0546\ \cdot 10^{-34})^2}{2 \cdot 9.11 \cdot 10^{-31}}\right)(3 \cdot 3.14159^2 \cdot 1.788 \cdot 10^{29})^{2/3} = 1.854 \cdot 10^{-18}\ \mathrm{J}$$

One joule is equal to 6.241509·10^{18} electron-volts:

Thus,

$$E_F = 1.854 \cdot 10^{-18}\ \mathrm{J} \cdot 6.241509 \cdot 10^{18} = 11.57\ \mathrm{eV}$$

The Fermi temperature T_F is the temperature associated with the Fermi energy by solving

$$E_F = kT_F$$

where k = 1.38·10^{-23} is Boltzmann's constant.

Thus,

$$T_F = \frac{E_F}{k} = \frac{1.854 \cdot 10^{-18}}{1.38 \cdot 10^{-23}} = 13.43 \cdot 10^4 \text{ K}$$

Answer. $E_F = 11.57 \text{ eV}$, $T_F = 13.43 \cdot 10^4 \text{ K}$.