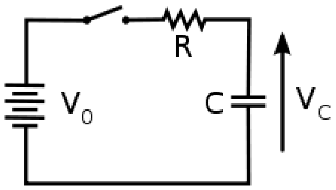
Answer on Question #40770, Physics, Electric Circuits

Dc supply is 10 volts. Describe how the current through the circuit and the voltage across the capacitor will develop when the switch is closed, and how this is affected by the resistor value

Answer



A series circuit containing only a resistor, a capacitor, a switch and a constant DC source of voltage V_0 is known as a charging circuit. If the capacitor is initially uncharged while the switch is open, and the switch is closed at t_0, it follows from Kirchhoff's voltage law that

$$V_0 = v_R(t) + v_C(t) = i(t)R + \frac{1}{C}\int_{t_0}^{t} i(s)ds.$$

Taking the derivative and multiplying by C, gives a first-order differential equation:

$$RC\frac{di(t)}{dt} + i(t) = 0.$$

At $t = 0$, the voltage across the capacitor is zero and the voltage across the resistor is V_0. The initial current is then $i(0) = \frac{V_0}{R}$. With this assumption, solving the differential equation yields

$$i(t) = \frac{V_0}{R}e^{-\frac{t}{\tau}},$$

$$v_C(t) = V_0(1 - e^{-\frac{t}{\tau}}),$$

where $\tau = RC$ is the time constant of the system. As the capacitor reaches equilibrium with the source voltage, the voltages across the resistor and the current through the entire circuit decay exponentially.

The voltage across the capacitor will asymptotically rise to V_0 when $t \to \infty$.

The value of R affects the time constant of a resistor-capacitor charging circuit (more R means slower charging; less R means faster charging).

$$i(t) = C\frac{dv_C}{dt}.$$

Current $i(t)$ is proportional to the rate of change of voltage $\frac{dv_C}{dt}$. Since current is inversely proportional to resistance in a circuit powered by a voltage source, so must be the capacitor charging rate.