Answer on Question #40734, Physics, Molecular Physics

One mole of oxygen at STP is adiabatically compressed to 5 atm. Calculate the final temperature. Also, calculate the work done on the gas. Take γ = 1.4 and R = 8.31 J mol^{-1} K^{-1}.

Solution:

Given:

Standard temperature and pressure (informally abbreviated as STP) is

T_1 = 273.15 K

p_1 = 100 kPa

p_2 = 5 atm =5· 101325 Pa = 506.625 kPa

T_1 = ?

An adiabatic process is one in which no heat is gained or lost by the system.

The adiabatic condition

$$pV^{\gamma} = const$$

where p is pressure, V is volume, and $\gamma = 1.4$ is the adiabatic index.

An ideal gas can be characterized by three state variables: absolute pressure (P), volume (V), and absolute temperature (T). The relationship between them may be deduced from kinetic theory and is called the ideal gas law:

$$pV = nRT$$

where n = number of moles, R = universal gas constant = 8.31 J/mol K.

Hence:

$$V = \frac{nRT}{p}$$

$$p\left(\frac{nRT}{p}\right)^{\gamma} = const$$

$$p^{1-\gamma}T^{\gamma} = const$$

Hence

$$p_1^{1-\gamma}T_1^{\gamma} = p_2^{1-\gamma}T_2^{\gamma}$$

$$T_2 = T_1\left(\frac{p_1^{1-\gamma}}{p_2^{1-\gamma}}\right)^{\frac{1}{\gamma}}$$

$$T_2 = 273.15 \cdot \left(\frac{100^{1-1.4}}{506.625^{1-1.4}}\right)^{\frac{1}{1.4}} = 434.25 \text{ K}$$

The work, **done by the gas** in adiabatic process is

$$W_b = -\alpha nRT_1\left(\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)$$

where α is the number of degrees of freedom divided by two.

α is 5/2 for diatomic gas such as oxygen.

Thus,

$$W_b = -\frac{5}{2} \cdot 1 \cdot 8.31 \cdot 273.15 \cdot \left(\left(\frac{506.625}{100} \right)^{\frac{1.4-1}{1.4}} - 1 \right) = -3346.86 \text{ J}$$

And work done on the gas

$$W = -W_b = 3.35 \text{ kJ}$$

Answer. $T_2 = 434.25$ K, $W = 3.35$ kJ.