

Answer on Question #40732, Physics, Mechanics | Kinematics | Dynamics

Consider a system of three equal mass particles moving in a plane; their positions are given by \mathbf{a}_i vectors and \mathbf{b}_i vectors.

For particle 1, $\mathbf{a}_1 = 3t^2 \mathbf{i} + 4 \mathbf{j}$

For particle 2, $\mathbf{a}_2 = 7t + 5 \mathbf{i} + 2 \mathbf{j}$

For particle 3, $\mathbf{a}_3 = 2t \mathbf{i} + 3t + 4 \mathbf{j}$

Determine the position and velocity of the center of mass as functions of time

Solution

For the 'a' component of the center of mass's position vector is R_a where:

$$R_a = \frac{\sum_i m_i \mathbf{a}_i}{\sum_i m_i} = \frac{m\mathbf{a}_1 + m\mathbf{a}_2 + m\mathbf{a}_3}{m + m + m} = \frac{1}{3}(\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3) = \frac{1}{3}(3t^2 + 4 + 7t + 5 + 2t) = t^2 + 3t + 3.$$

The 'a' component of the center of mass's velocity, V_a is found by differentiating with respect to time

$$V_a = \frac{dR_a}{dt} = 2t + 3.$$

For the 'b' component of the center of mass's position vector is R_b where:

$$R_b = \frac{\sum_i m_i \mathbf{b}_i}{\sum_i m_i} = \frac{m\mathbf{b}_1 + m\mathbf{b}_2 + m\mathbf{b}_3}{m + m + m} = \frac{1}{3}(\mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3) = \frac{1}{3}(2 + 3t + 4) = t + 2.$$

The 'b' component of the center of mass's velocity, V_b is found by differentiating with respect to time

$$V_b = \frac{dR_b}{dt} = 1.$$

The position of the center of mass:

$$\vec{R} = (t^2 + 3t + 3)\vec{i} + (t + 2)\vec{j}.$$

The velocity of the center of mass:

$$\vec{V} = (2t + 3)\vec{i} + \vec{j}.$$