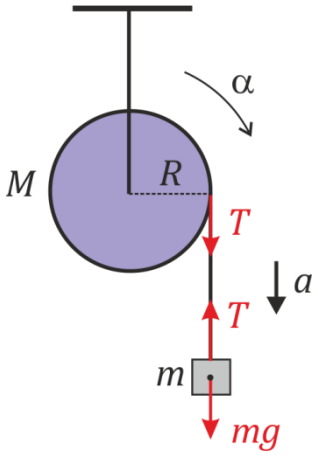
Answer on Question #40696, Physics, Mechanics | Kinematics | Dynamics

A light thread with a mass m tied to its end is wound over a uniform solid cylinder of radius R and mass M and the system is set into motion at t = 0.

Obtain the angular velocity of the cylinder.

Solution:



Method 1:

Sum of the torques is equal to mass moment of inertia (*I*) times angular acceleration α

$$mgR = \left(\frac{MR^2}{2} + mR^2\right)\alpha$$

So,

$$\alpha = \frac{2mg}{\left(\frac{M}{2} + m\right)R}$$

The angular velocity is

$$\omega = \alpha t = \frac{2mgt}{\left(\frac{M}{2} + m\right)R} = \frac{gt}{\left(1 + \frac{M}{2m}\right)R}$$

Method 2:

The equation of motion is

$$mg - T = ma$$

$$M = I\alpha$$

where torque $M = TR$, momentum of inertia $I = \frac{MR^2}{2}$, linear acceleration $a = R\alpha$.

From these equations we obtain:

$$TR = \frac{MR^2}{2}\alpha$$

$$T = \frac{MR}{2}\alpha$$

$$mg - \frac{MR}{2}\alpha = mR\alpha$$

For angular acceleration

$$\alpha = \frac{2mg}{\left(\frac{M}{2} + m\right)R}$$

The angular velocity is

$$\omega = \alpha t = \frac{2mgt}{\left(\frac{M}{2} + m\right)R} = \frac{gt}{\left(1 + \frac{M}{2m}\right)R}$$

Answer. $\omega = \frac{gt}{\left(1+\frac{M}{2m}\right)R}$