Answer on Question#40690 – Physics – Mechanics

A body of mass 0.5 kg travels in a straight line with velocity V = ax3/2 where a = 5 m1/2s1. Find the work done by the net force during its displacement from x = 0 to x = 2 m is :-

(1)1.5 J

(2)50 J

(3)10 J

(4)100 J

Solution:

$\mathrm{m} = 0.5\mathrm{kg} - \text{mass of the body};$

Here the unit of the constant a not correct, a should have dimension $\mathrm{L}^{\frac{1}{2}}\,\mathrm{T}^{-1}$ and not LT^{-2}.

Velocity is given by ($\mathrm{a} = 5\frac{\mathrm{m}}{\mathrm{s}^2}$):

$$\mathrm{v} = \frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{ax}^{\frac{3}{2}} \quad (1)$$

Let $\mathrm{a}' = $ acceleration:

$$\mathrm{a}' = \frac{\mathrm{dv}}{\mathrm{dt}} \Longrightarrow \mathrm{a}' = \frac{\mathrm{d}\left(\mathrm{ax}^{\frac{3}{2}}\right)}{\mathrm{dt}} = \frac{3\mathrm{a}}{2}\mathrm{x}^{\frac{1}{2}}\frac{\mathrm{dx}}{\mathrm{dt}} \quad (2)$$

$$(1)\text{to } (2):$$

$$\mathrm{a}' = \frac{3\mathrm{a}}{2}\mathrm{x}^{\frac{1}{2}}\mathrm{ax}^{\frac{3}{2}} = \frac{3\mathrm{a}^2}{2}\mathrm{x}^3 \quad (3)$$

Newton's second Law for the body:

$$\mathrm{F} = \mathrm{ma}' \quad (4)$$

$$(3)\text{to}(4):$$

$$\mathrm{F} = \mathrm{m}\frac{3\mathrm{a}^2}{2}\mathrm{x}^3 \quad (5)$$

The work done by the net force during its displacement:

$$\mathrm{W} = \int_{\mathrm{x}=\mathrm{x}_1}^{\mathrm{x}=\mathrm{x}_2} \mathrm{F}\mathrm{dx} \quad (6)$$

$$(5)\text{to}(6):$$

$$\mathrm{W} = \int_{\mathrm{x}=\mathrm{x}_1}^{\mathrm{x}=\mathrm{x}_2} \mathrm{m}\frac{3\mathrm{a}^2}{2}\mathrm{x}^3\mathrm{dx} = \mathrm{m}\frac{3\mathrm{a}^2}{2}\cdot\frac{\mathrm{x}^4}{4}\Big|_{\mathrm{x}=0}^{\mathrm{x}=2} = \frac{3\mathrm{a}^2\mathrm{m}}{2}\left(\frac{2^4}{4} - 0\right) = 6\mathrm{a}^2\mathrm{m}$$

$$= 6\cdot\left(5\frac{\mathrm{m}}{\mathrm{s}^2}\right)^2\cdot 0.5\mathrm{kg} = 75\mathrm{J}$$

Answer: work done by the net force is equal to 75J